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Problem 1. Atmospheric refraction around a planet (50 points)

The light refraction in the atmosphere of a planet is an optical effect, which is due to the optical inhomogeneity of the atmosphere, thus the light propagation obey the Fermat principle. In this problem you have to find out if it is the possible that due to the atmospheric refraction an astronaut could see his back. The atmospheric absorption is negligible and the structure of the atmosphere is stable**.** Here you have some theoretical issues to help you to solve the problem:

The situation you have to investigate is sketched in the figure 1.

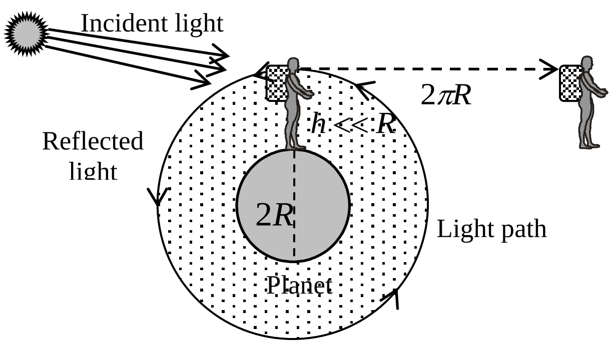
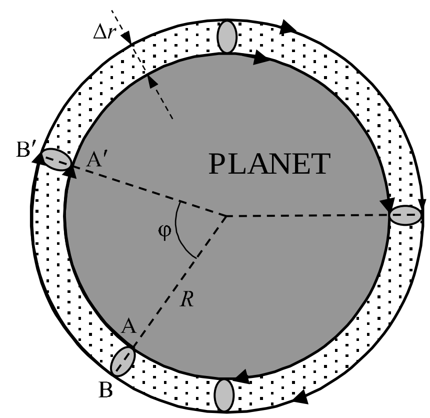


Fig. 1



The non-homogeneity of the atmosphere can be simulated by considering the atmosphere consisted of concentric spherical thin layers of thickness Δr. In one layer the refractive index is $n > 1$ due to the presence of the gas molecules. The refractive index of vacuum is $n_{vacuum} = 1$

The plane wave-front of the light AB has to remain normal on the surface of the planet and at the same distance from it, despite that the propagation occurs through an inhomogeneous atmosphere consisted of concentric spherical thin, homogenous layers.

The consequence of the model is that near the planet surface the following relationship occurred

$$n - 1 = \alpha N,$$

whith N the gas molecules concentration, and α a constant.

For Earth atmosphere near the surface:

$$n - 1 \approx 11.35 \cdot 10^{-5}.$$

In the table are given the data for 10 planets (8 from Solar System and 2 exoplanets).

$R-$planet radius in m; $m-$the mass of on gas molecule of the dominant one near the surface of the planet in atomic mass unit (1 a.m.u. $= 1.66 \cdot 10^{-27}$ kg); $p-$atmospheric pressure near the surface of the planet in bar (1 bar = 10^5 Pa = 10^5 Nm^{-2}); $T-$atmosphere temperature near the surface of the planet in K; $g-$gravitational acceleration in ms^{-2}; You will assume that the molecules are spheres with an effective diameter d. The Boltzman-constant is $k = 1.38 \cdot 10^{-23}$ J/K.

Planet	R 10^5 m	The gas	m amu	p 10^5 Pa	T K	g m/s^2	d m
Mercur	24	CO_2	44	10^{-14}	450	3,7	$3{,}75\cdot10^{-10}$
Venus	62	CO_2	44	100	700	8,5	$3{,}75\cdot10^{-10}$
Earth	64	N_2	28	1	300	9,8	$3\cdot10^{-10}$
Mars	34	CO_2	44	10^{-2}	200	3,7	$3{,}75\cdot10^{-10}$
Jupiter	715	H_2	2	1	340	23	$2\cdot10^{-10}$
Saturn	600	H_2	2	1000	400	10,4	$2\cdot10^{-10}$
Uranus	260	H_2	2	1000	50	8,8	$2\cdot10^{-10}$
Neptun	250	H_2	2	1000	70	11,1	$2\cdot10^{-10}$
Exoplanet X	72,5	H_2	28	15	500	20	$2\cdot10^{-10}$
Exoplanet Y	25,75	CH_4	16	20	600	15	$4\cdot10^{-10}$

By using the data, you have to analyze and justify the possibility of the occurrence of the described phenomenon for each planet, considering the propagation of the light in the near the planet surface layer of the atmosphere.

a) Derive the necessary geometrical condition for producing the light refraction around a planet, if Δn is the small variance of the refractive index of the gas medium correlated with the small gas layer thickness Δr

b) Express the previous condition in terms of the thermodynamic parameters of the gas near the surface of the atmosphere i.e. the pressure, the temperature, the concentration of the gas molecules, the mass of a molecule, by calculating the value of coefficient α for the gas in the layer near the Earth surface. The refractive index n of the gas, near the surface of each planet, obey this relation $\frac{\Delta n}{n}=\frac{\alpha\Delta N}{n}\approx\alpha N$

c) Calculate the numerical values of those parameters which allow you to establish the possibility of the occurrence of the described optical phenomenon. Around which planets the phenomenon occurs. A table which centralize the data is an advantage for better understanding the solution.

Problem 1. Marking scheme   Atmospheric refraction around a planet (50 points)

-	a) Any relations which drive to the correct answer	**10 points**
-	b)	**20 points**
	○ thermodynamic analysis	10 points
	○ the value and signification of α	10 points
-	c)	**20 points**
	○ calculations for each planets	15 points
	○ correct identification of the planets	5 puncte

a) The optical condition for the light refraction in the planet atmosphere 10 points

As seen in the figure 1the plane wave front AB has to remain perpendicular to the surface of the planet, and at the same distance from that. If we consider the thin layer near the planet which can be considered homogenous the time of light travel will be the same on the circle sectors AA′ și BB′ corresponding to the same angle φ

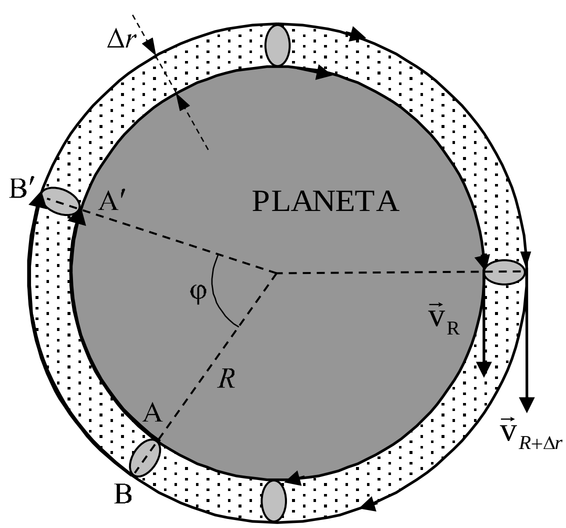


Fig.1

$$t = \frac{AA'}{v_R} = \frac{BB'}{v_{R+\Delta r}},$$

v_R - speed of light in the thin spherical homogenous layer near the planet surface – radius R

$v_{R+\Delta r}$ speed of light in the thin spherical homogenous layer near the planet surface – radius $R+\Delta r$;

$$v_{R+\Delta r} > v_R;$$

$$n_R = \frac{c}{v_R};\ n_{R+\Delta r} = \frac{c}{v_{R+\Delta r}} < n_R,$$

c speed of light in vacuum,
n_R refraction index of the atmosphere layer – radius R
$n_{R+\Delta r}$ refraction index of the atmosphere layer – radius $R+\Delta r$.

Results:

$$AA'\cdot n_R = BB'\cdot n_{R+\Delta r};\ n_{R+\Delta r} < n_R;$$
$$AA' = R\varphi;\ BB' = (R+\Delta r)\varphi;$$
$$Rn_R = (R+\Delta r)n_{R+\Delta r};\ n_{R+\Delta r} = n_R + \Delta n,$$

where $\Delta n < 0$ is the variance of the refraction index of the atmospheric gas associated with the height Δr ;

$$Rn_R = (R+\Delta r)\cdot(n_R+\Delta n);$$
$$\Delta r\cdot\Delta n \approx 0;$$
$$-\frac{\Delta n}{n_R\Delta r} = \frac{1}{R};$$
$$n_R = n;\ -\frac{\Delta n}{n\Delta r} = \frac{1}{R};\ \frac{-\frac{\Delta n}{n}}{\Delta r} = \frac{1}{R};\ -\frac{\Delta n}{n\Delta r} = \frac{1}{R};$$

The condition of the occurrence of the refraction around the planet

$$-\frac{\Delta n/n}{\Delta r} = \frac{1}{R},$$

Using the following notations:

- theoretical curvature of the Earth surface $C_{\text{suprafata planeta}} = \frac{1}{R};$
- the curvature of the light ray in the atmosphere layer near the Earth surface $C_{light} = -\frac{\Delta n}{n\Delta r};$

Thus:

$$C_{light} = C_{\text{supraface}\,of\,the\,\text{planet}}$$

b1) Relationship between the refractive index and the physical properties of the gas near the planet surface (pressure, temperature, molecule concentration, mass of the molecules). 10 points

According with the notations in figure 1 it can be obtain the equilibrium condition for the gas column near the Earth's surface.

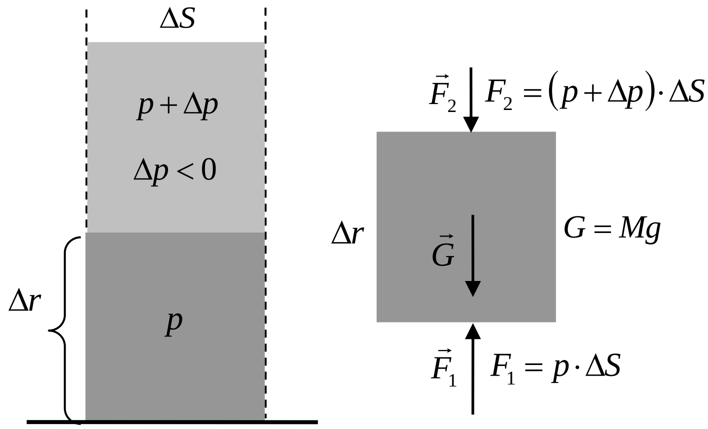


Fig. 1

$$\vec{F}_1+\vec{G}+\vec{F}_2=0;$$
$$F_1-G-F_2=0;$$
$$p\cdot\Delta S-Mg-(p+\Delta p)\cdot\Delta S=0;$$
$$p\cdot\Delta S-Mg-p\cdot\Delta S-\Delta p\cdot\Delta S=0;$$
$$-\Delta p\cdot\Delta S=Mg;$$
$$-\Delta p\cdot\Delta S=Nmg\cdot\Delta r\cdot\Delta S;$$
$$-\Delta p=Nmg\cdot\Delta r,$$

where m is the mass of a molecule, and g is the gravitational acceleration at the base of the atmosphere:

$$p=NkT;\quad \Delta p=kT\Delta N;$$
$$-kT\cdot\Delta N=Nmg\cdot\Delta r;$$

$$-\frac{\Delta N}{\Delta r}=N\frac{mg}{kT}$$ 10 points

According to the relationship

$n-1=\alpha N,$

And the fact that the $n_{\text{vid}}=1$ the coefficient α represents the volume.

$\langle N\rangle_{\text{SI}}=\text{m}^{-3};\ \langle\alpha\rangle_{\text{SI}}=\text{m}^{3}$ and $n>1$ being proportional to the concentration of the molecules in the atmosphere.

$$n-1=\alpha N,$$
$$\Delta(n-1)=\Delta(\alpha N);\ \Delta n=\alpha\Delta N;$$
$$\frac{\Delta n}{n}=\frac{\alpha\Delta N}{n};\ \ n\approx 1;$$
$$\frac{\Delta n}{n}=\frac{\alpha\Delta N}{n}\approx\alpha\Delta N,$$

Using the relation (1) :

$$\frac{\Delta n}{n} = \alpha\Delta N;\ \Delta N = \frac{\Delta n}{\alpha n};$$

$$-\frac{\Delta N}{\Delta r} = N\frac{mg}{kT}$$

$$-\frac{\frac{\Delta n}{\alpha n}}{\Delta r} = N\frac{mg}{kT};\ -\frac{\Delta n}{\alpha n \Delta r} = N\frac{mg}{kT};$$

The relationship between the light curvature and the physical properties of the atmosphere will be:

$$C_{light} = -\frac{\Delta n}{n\Delta r} = \alpha N\frac{mg}{kT} \quad (2)$$

This one can be experimental determined for each planet atmosphere and compared with the each planet surface curvature.

$$C_{light,\ \mathrm{experimental}} = \alpha N\frac{mg}{kT};\ C_{planet} = \frac{1}{R},$$

The condition is

$$C_{\mathrm{light}} = C_{planet};\ (3)$$

b2) The value of α, corelated with the volume of the gas molecule 10 points

α reprezents the volume of a gas molecule.

For Earth :

$$n - 1 \approx 11{,}35\cdot 10^{-5} = \alpha N;\ p = 1\,\mathrm{bar} = 10^5\ \mathrm{Nm^{-2}};\ T = 300\ \mathrm{K},$$

so

$$N = \frac{p}{kT} = \frac{10^5\ \mathrm{Nm^{-2}}}{1{,}38\cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}\cdot 300\,\mathrm{K}},$$

where $k = 1{,}38 \cdot 10^{-23}$ J/K (constanta lui Boltzman);

$$N \approx 2{,}41\cdot 10^{25}\ \mathrm{m^{-3}},$$

Is the nytrogen concentration in the Earth atmosphere;

$$11{,}35\cdot 10^{-5} = \alpha \cdot 2{,}41\cdot 10^{25}\ \mathrm{m^{-3}};$$

$$\alpha \approx 4{,}70\cdot 10^{-30}\ \mathrm{m^3},$$

The volume of an nytrogen molecule with the diameter $d_{\mathrm{N_2}} \approx 3\cdot 10^{-10}\ \mathrm{m}$ is:

$$V_{1,\mathrm{N_2}} = \frac{4\pi}{3}\left(\frac{d_{\mathrm{N_2}}}{2}\right)^3 = \frac{4\cdot 3{,}14}{3}\cdot\frac{9\cdot 10^{-30}}{8}\ \mathrm{m^3};$$

Thus

$$V_{1,N_2} \approx 4{,}71 \cdot 10^{-30}\ \mathrm{m}^3 \approx \alpha.$$

c) Data analysis 20 points

- For Earth atmosphere considered full of nitrogen where p = 1 bar and temperature T = 300 K results:

$$N = \frac{p}{kT} = \frac{10^5\ \mathrm{Nm}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 300\ \mathrm{K}};$$

$$N \approx 2{,}41 \cdot 10^{25}\ \mathrm{m}^{-3};$$

$$V_{1,N_2} = \frac{4\pi}{3}\left(\frac{d_{N_2}}{2}\right)^3 = \frac{4 \cdot 3{,}14}{3} \cdot \frac{9 \cdot 10^{-30}}{8}\ \mathrm{m}^3;$$

$$V_{1,N_2} \approx 4{,}71 \cdot 10^{-30}\ \mathrm{m}^3 \approx \alpha;$$

$$\alpha N = 4{,}71 \cdot 10^{-30}\ \mathrm{m}^{-3} \cdot 2{,}41 \cdot 10^{25}\ \mathrm{m}^3 = 11{,}35 \cdot 10^{-5};$$

$$\alpha N \cdot \frac{mg}{kT} = 11{,}35 \cdot 10^{-5} \cdot \frac{28 \cdot 1{,}66 \cdot 10^{-27}\ \mathrm{kg} \cdot 9{,}8\ \mathrm{ms}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 3 \cdot 10^2\ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 0{,}12 \cdot 10^{-7}\ \frac{1}{\mathrm{m}}.$$

- For Venus atmosphere $T \approx 700$ K, $p \approx 100$ atm, gas CO_2:

$$N = \frac{p}{kT} = \frac{10^2 \cdot 10^5\ \mathrm{Nm}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 700\ \mathrm{K}};$$

$$N \approx 10^{27}\ \mathrm{m}^{-3}.$$

CO_2 molecule diameter

$$d_{CO_2} = 1{,}25 \cdot d_{N_2},$$

CO_2 molecule volume

$$d_{CO_2} = 3{,}75 \cdot 10^{-10}\ \mathrm{m};$$

$$V_{1,CO_2} = \frac{4\pi}{3}\left(\frac{d_{CO_2}}{2}\right)^3 = \frac{4 \cdot 3{,}14}{3} \cdot \frac{(3{,}75)^3 \cdot 10^{-30}}{8}\ \mathrm{m}^3;$$

$$V_{1,CO_2} = 27{,}59 \cdot 10^{-30}\ \mathrm{m}^3 = \alpha,$$

$$\alpha N = 27{,}59 \cdot 10^{-30}\ \mathrm{m}^{-3} \cdot 10^{27}\ \mathrm{m}^3 = 27{,}59 \cdot 10^{-3};$$

$$\alpha N \cdot \frac{mg}{kT} = 27{,}59 \cdot 10^{-3} \cdot \frac{44 \cdot 1{,}66 \cdot 10^{-27}\ \mathrm{kg} \cdot 8{,}5\ \mathrm{ms}^{-2}}{1{,}38 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \cdot 7 \cdot 10^2\ \mathrm{K}};$$

$$\alpha N\cdot\frac{mg}{kT}\approx 17{,}7318\cdot10^{-7}\ \mathrm{m}^{-1}.$$

-For ***Mars*** :

$$N=\frac{p}{kT}=\frac{10^{-2}\cdot10^{5}\ \mathrm{Nm}^{-2}}{1{,}38\cdot10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}\cdot 200\ \mathrm{K}};$$

$$N\approx 3{,}62\cdot10^{23}\ \mathrm{m}^{-3};$$

$$V_{1,\mathrm{CO_2}}=27{,}59\cdot10^{-30}\ \mathrm{m}^{3}=\alpha;$$

$$\alpha N=27{,}59\cdot10^{-30}\ \mathrm{m}^{-3}\cdot 3{,}62\cdot10^{23}\ \mathrm{m}^{3}=99{,}87\cdot10^{-7};$$

$$\alpha N\cdot\frac{mg}{kT}=99{,}87\cdot10^{-7}\cdot\frac{44\cdot1{,}66\cdot10^{-27}\ \mathrm{kg}\cdot 3{,}7\ \mathrm{ms}^{-2}}{1{,}38\cdot10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}\cdot 2\cdot10^{2}\ \mathrm{K}};$$

$$\alpha N\cdot\frac{mg}{kT}\approx 9{,}77886\cdot10^{-10}\ \mathrm{m}^{-1}.$$

- For Jupiter :

$$N=\frac{p}{kT}=\frac{10^{5}\ \mathrm{Nm}^{-2}}{1{,}38\cdot10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}\cdot 340\ \mathrm{K}};$$

$$N\approx 2{,}13\cdot10^{25}\ \mathrm{m}^{-3};$$

$$V_{1,\mathrm{H_2}}=4{,}18\cdot10^{-30}\ \mathrm{m}^{3}=\alpha;$$

$$\alpha N=4{,}18\cdot10^{-30}\ \mathrm{m}^{-3}\cdot 2{,}13\cdot10^{25}\ \mathrm{m}^{3}=8{,}9\cdot10^{-5};$$

$$\alpha N\cdot\frac{mg}{kT}=8{,}9\cdot10^{-5}\cdot\frac{2\cdot1{,}66\cdot10^{-27}\ \mathrm{kg}\cdot 23\ \mathrm{ms}^{-2}}{1{,}38\cdot10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}\cdot 340\ \mathrm{K}};$$

$$\alpha N\cdot\frac{mg}{kT}\approx 14{,}48\cdot10^{-10}\ \mathrm{m}^{-1}.$$

- For ***Uranus***:

$$N=\frac{p}{kT}=\frac{10^{3}\cdot10^{5}\ \mathrm{Nm}^{-2}}{1{,}38\cdot10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}\cdot 50\ \mathrm{K}};$$

$$N\approx 145\cdot10^{27}\ \mathrm{m}^{-3};$$

$$V_{1,\mathrm{H_2}}=4{,}18\cdot10^{-30}\ \mathrm{m}^{3}=\alpha;$$

$$\alpha N=4{,}18\cdot10^{-30}\ \mathrm{m}^{-3}\cdot 145\cdot10^{27}\ \mathrm{m}^{3}=606{,}1\cdot10^{-3};$$

$$\alpha N\cdot\frac{mg}{kT}=606{,}1\cdot10^{-3}\cdot\frac{2\cdot1{,}66\cdot10^{-27}\ \mathrm{kg}\cdot 8{,}8\ \mathrm{ms}^{-2}}{1{,}38\cdot10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}\cdot 50\ \mathrm{K}};$$

$$\alpha N\cdot\frac{mg}{kT}\approx 2566{,}35\cdot10^{-8}\ \mathrm{m}^{-1}.$$

- For ***Saturn***:

$$N = \frac{p}{kT} = \frac{10^3 \cdot 10^5 \ \mathrm{Nm^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 400 \ \mathrm{K}};$$

$$N \approx 18 \cdot 10^{27} \ \mathrm{m^{-3}};$$

$$V_{1,\mathrm{H_2}} = 4{,}18 \cdot 10^{-30} \ \mathrm{m^3} = \alpha;$$

$$\alpha N = 4{,}18 \cdot 10^{-30} \ \mathrm{m^{-3}} \cdot 18 \cdot 10^{27} \ \mathrm{m^3} = 75{,}24 \cdot 10^{-3};$$

$$\alpha N \cdot \frac{mg}{kT} = 75{,}24 \cdot 10^{-3} \cdot \frac{2 \cdot 1{,}66 \cdot 10^{-27} \ \mathrm{kg} \cdot 10{,}4 \ \mathrm{ms^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 400 \ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 470{,}63 \cdot 10^{-9} \ \mathrm{m^{-1}}.$$

- For ***Mercury***:

$$N = \frac{p}{kT} = \frac{10^{-14} \cdot 10^5 \ \mathrm{Nm^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 450 \ \mathrm{K}};$$

$$N \approx 16 \cdot 10^{10} \ \mathrm{m^{-3}};$$

$$V_{1,\mathrm{H_2}} = 27{,}59 \cdot 10^{-30} \ \mathrm{m^3} = \alpha;$$

$$\alpha N = 27{,}59 \cdot 10^{-30} \ \mathrm{m^{-3}} \cdot 16 \cdot 10^{10} \ \mathrm{m^3} = 441{,}44 \cdot 10^{-20};$$

$$\alpha N \cdot \frac{mg}{kT} = 441{,}44 \cdot 10^{-20} \cdot \frac{44 \cdot 1{,}66 \cdot 10^{-27} \ \mathrm{kg} \cdot 3{,}7 \ \mathrm{ms^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 450 \ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 1921 \cdot 10^{-25} \ \mathrm{m^{-1}}.$$

- For ***Neptun***:

$$N = \frac{p}{kT} = \frac{10^3 \cdot 10^5 \ \mathrm{Nm^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 70 \ \mathrm{K}};$$

$$N \approx 10^{29} \ \mathrm{m^{-3}};$$

$$V_{1,\mathrm{H_2}} = 4{,}18 \cdot 10^{-30} \ \mathrm{m^3} = \alpha;$$

$$\alpha N = 4{,}18 \cdot 10^{-30} \ \mathrm{m^{-3}} \cdot 10^{29} \ \mathrm{m^3} = 4{,}18 \cdot 10^{-1};$$

$$\alpha N \cdot \frac{mg}{kT} = 4{,}18 \cdot 10^{-1} \cdot \frac{2 \cdot 1{,}66 \cdot 10^{-27} \ \mathrm{kg} \cdot 11{,}1 \ \mathrm{ms^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 70 \ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 15{,}95 \cdot 10^{-6} \ \mathrm{m^{-1}}.$$

- For exoplanet ***X***:

$$N = \frac{p}{kT} = \frac{15 \cdot 10^5 \ \mathrm{Nm^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 500 \ \mathrm{K}};$$

$$N \approx 2{,}17 \cdot 10^{26} \ \mathrm{m^{-3}};$$

$$V_{1,\mathrm{N_2}} = 4{,}71 \cdot 10^{-30} \ \mathrm{m^3} = \alpha;$$

$$\alpha N = 4{,}71 \cdot 10^{-30} \ \mathrm{m^{-3}} \cdot 2{,}17 \cdot 10^{26} \ \mathrm{m^3} = 10{,}22 \cdot 10^{-4};$$

$$\alpha N \cdot \frac{mg}{kT} = 10{,}22 \cdot 10^{-4} \cdot \frac{28 \cdot 1{,}66 \cdot 10^{-27} \ \mathrm{kg} \cdot 20 \ \mathrm{ms^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 500 \ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 137{,}68 \cdot 10^{-9} \ \mathrm{m^{-1}}.$$

- For exoplanet ***Y***:

$$N = \frac{p}{kT} = \frac{20 \cdot 10^5 \ \mathrm{Nm^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 600 \ \mathrm{K}};$$

$$N \approx 2{,}41 \cdot 10^{26} \ \mathrm{m^{-3}};$$

$$V_{1,\mathrm{CH_4}} = \frac{4\pi}{3}\left(\frac{d_{\mathrm{CH_4}}}{2}\right)^3 = \frac{4 \cdot 3{,}14}{3} \cdot \frac{(4)^3 \cdot 10^{-30}}{8} \ \mathrm{m^3};$$

$$V_{1,\mathrm{H_2}} = 33{,}5 \cdot 10^{-30} \ \mathrm{m^3} = \alpha;$$

$$\alpha N = 33{,}5 \cdot 10^{-30} \ \mathrm{m^{-3}} \cdot 2{,}41 \cdot 10^{26} \ \mathrm{m^3} = 80{,}7 \cdot 10^{-4};$$

$$\alpha N \cdot \frac{mg}{kT} = 80{,}7 \cdot 10^{-4} \cdot \frac{16 \cdot 1{,}66 \cdot 10^{-27} \ \mathrm{kg} \cdot 15 \ \mathrm{ms^{-2}}}{1{,}38 \cdot 10^{-23} \ \frac{\mathrm{J}}{\mathrm{K}} \cdot 600 \ \mathrm{K}};$$

$$\alpha N \cdot \frac{mg}{kT} \approx 3883 \cdot 10^{-10} \ \mathrm{m^{-1}}.$$

Centralized rezults.

Planeta	p bar	m uam	T K	g $\mathrm{m/s^2}$	N $\mathrm{m^{-3}}$	α $\mathrm{m^3}$	$\alpha Nmg/kT$ $\mathrm{m^{-1}}$
Pământ	1	28	300	9,8	$2{,}41 \cdot 10^{25}$	$4{,}71 \cdot 10^{-30}$	$0{,}12 \cdot 10^{-7}$
Venus	100	44	700	8,5	10^{27}	$27{,}59 \cdot 10^{-30}$	$17{,}73 \cdot 10^{-7}$
Marte	10^{-2}	44	200	3,7	$3{,}62 \cdot 10^{23}$	$27{,}59 \cdot 10^{-30}$	$9{,}7 \cdot 10^{-10}$

Jupiter	1	2	340	23	$2{,}13\cdot10^{25}$	$4{,}18\cdot10^{-30}$	$14{,}48\cdot10^{-10}$
Uranus	1000	2	50	8,8	$145\cdot10^{27}$	$4{,}18\cdot10^{-30}$	$2566{,}35\cdot10^{-8}$
Saturn	1000	2	400	10,4	$18\cdot10^{27}$	$4{,}18\cdot10^{-30}$	$470{,}63\cdot10^{-9}$
Mercur	10^{-14}	44	450	3,7	$16\cdot10^{10}$	$27{,}59\cdot10^{-30}$	$1921\cdot10^{-25}$
Neptun	1000	2	70	11,1	10^{29}	$4{,}18\cdot10^{-30}$	$15{,}95\cdot10^{-6}$
Exo X	15	28	500	20	$2{,}17\cdot10^{26}$	$4{,}71\cdot10^{-30}$	$137{,}68\cdot10^{-9}$
Exo Y	20	16	600	15	$2{,}41\cdot10^{26}$	$33{,}5\cdot10^{-30}$	$388{,}3\cdot10^{-9}$

Planet	$\alpha Nmg / kT$ m^{-1}	R m	$1/R$ m^{-1}	$C_{light,\ \exp}$? $C_{\text{surface planet}}$
Earth	$0{,}12\cdot10^{-7}$	$64\cdot10^{5}$	$1{,}56\cdot10^{-7}$	$C_{light,\ \exp} < C_{\text{surface planet}}$
Venus	$17{,}73\cdot10^{-7}$	$62\cdot10^{5}$	$1{,}61\cdot10^{-7}$	$C_{light,\ \exp} > C_{\text{surface planet}}$
Marte	$9{,}7\cdot10^{-10}$	$34\cdot10^{5}$	$2{,}94\cdot10^{-6}$	$C_{light,\ \exp} < C_{\text{surface planet}}$
Jupiter	$14{,}48\cdot10^{-10}$	$715\cdot10^{5}$	$1{,}3\cdot10^{-8}$	$C_{light,\ \exp} < C_{\text{surface planet}}$
Uranus	$2566{,}35\cdot10^{-8}$	$26\cdot10^{6}$	$3{,}8\cdot10^{-8}$	$C_{light,\ \exp} > C_{\text{surface planet}}$
Saturn	$470{,}63\cdot10^{-9}$	$60\cdot10^{6}$	$1{,}6\cdot10^{-9}$	$C_{light,\ \exp} > C_{\text{surface planet}}$
Mercur	$1921\cdot10^{-25}$	$2{,}4\cdot10^{6}$	$416\cdot10^{-9}$	$C_{light,\ \exp} > C_{\text{surface planet}}$
Neptun	$15{,}95\cdot10^{-6}$	$25\cdot10^{6}$	$4\cdot10^{-9}$	$C_{light,\ \exp} > C_{\text{surface planet}}$
Exo X	$137{,}68\cdot10^{-9}$	$72{,}5\cdot10^{5}$	$137\cdot10^{-9}$	$C_{light,\ \exp} \cong C_{\text{surface planet}}$
Exo Y	$388{,}3\cdot10^{-9}$	$25{,}75\cdot10^{5}$	$388\cdot10^{-9}$	$C_{light,\ \exp} \cong C_{\text{surface planet}}$

e) Conclusion

- For all the Solar System planets

$$C_{light,\ \exp} \neq C_{\text{surface planet}}$$

Thus the condition is not fullfiled

- For the exoplanets X and Y

$$C_{\text{raza lumina, exp}} \approx C_{\text{suprafata planeta}},$$

Thus the astronaut can see its back

Problem 2 Black Hole in Milky Way

The astronomers have been surprised by some observational facts:

1) The intensity of the light coming from the center of the Milky Way is smaller than the intensity of the light coming from the stars around our galaxy;
2) The stars located close to the center of the Milky Way change its apparent position faster than those located far away from the center of the galaxy.
   To justify those observational facts, the scientists admit the presence of a black-hole in the center of Milky Way.

In the center of Milky Way, is located the hypothetical black-hole (Sagittarius A*). A star S* is orbiting the black-hole A*

In the table 1 the following data is presented : the date and the angular position coordinates $(\alpha;\beta)$ of the star S* at different moments of the observation. The coordinates represent the angular distances of the projection or the star S* in the coordinates system (U,W) , centered on the Sagittarius A* see figure 1.

An angular distance $\varphi = 1\,\text{arcsec}$ corresponds a linear distance in the sky -plane $d = 41\,\text{light day}$, so to a scale $S_0 = \dfrac{d}{\varphi} = 41\,\dfrac{\text{light day}}{\text{arcsec}}$.

	Date (year)	$\alpha(\text{arcsec})$	$\beta(\text{arcsec})$
1	1995.222	0.117	- 0.166
2	1997.526	0.097	- 0.189
3	1998.326	0.087	- 0.192
4	1999.041	0.077	- 0.193
5	2000.414	0.052	- 0.183
6	2001.169	0.036	- 0.167
7	2002.831	- 0.000	- 0.120
8	2003.584	- 0.016	- 0.083
9	2004.165	- 0.026	- 0.041
10	2004.585	- 0.017	0.008
11	2004.655	- 0.004	0.014
12	2004.734	0.008	0.017
13	2004.839	0.021	0.012
14	2004.936	0.037	0.009
15	2005.503	0.072	- 0.024
16	2006.041	0.088	- 0.050
17	2007.060	0.108	- 0.091

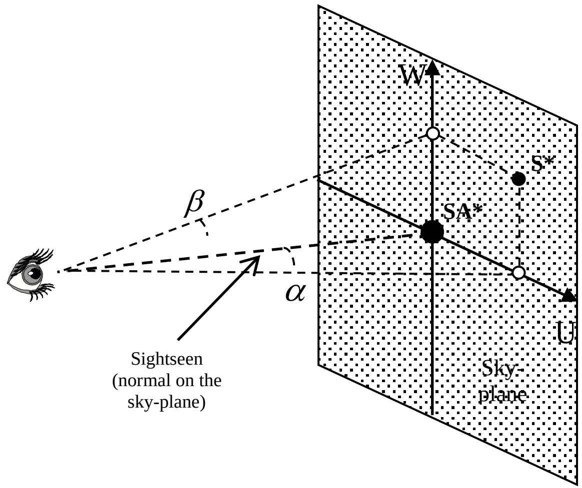


Fig. 1

By using the information provided your tasks are:

a) Plot the projection of the trajectory of the star S* in the plane P (see the figure 2). This plane is close to the observer. In this plane to correspond a linear distance d_0= 1200 mm so the scale is You have to use the milimetric paper, carbon copy sheet of paper and the transparent sheets for an accurate plot.

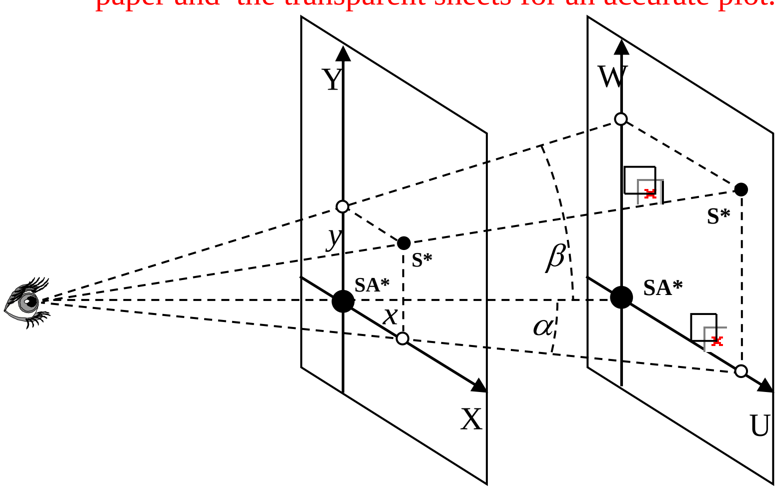


Fig. 2

b) By using the plot prove that the sight seen direction is normal to the plane of the orbit

c) Using your plot find out following elements of the real orbit of star S* around the black hole Sagittarius A*:

I. a – large semi-axis (in light days units) ; b – small semi-axis in (in light days units); eccentricity;

II. the minimum distance between S* and SA*(in light days units); r_{max} – the maximum distance between S* and SA* (in light days units);

III. The distance from the observer to the S*;

IV. the orbiting period of star S* around Sagittarius A* (will be accepted as a correct result the arithmetic mean of the maximum number of findings);

V. the total mass of the system "Sagitarius A* - Star S*".

Presenting the intermediate and final data in tables is recommended for an accurate evaluation.

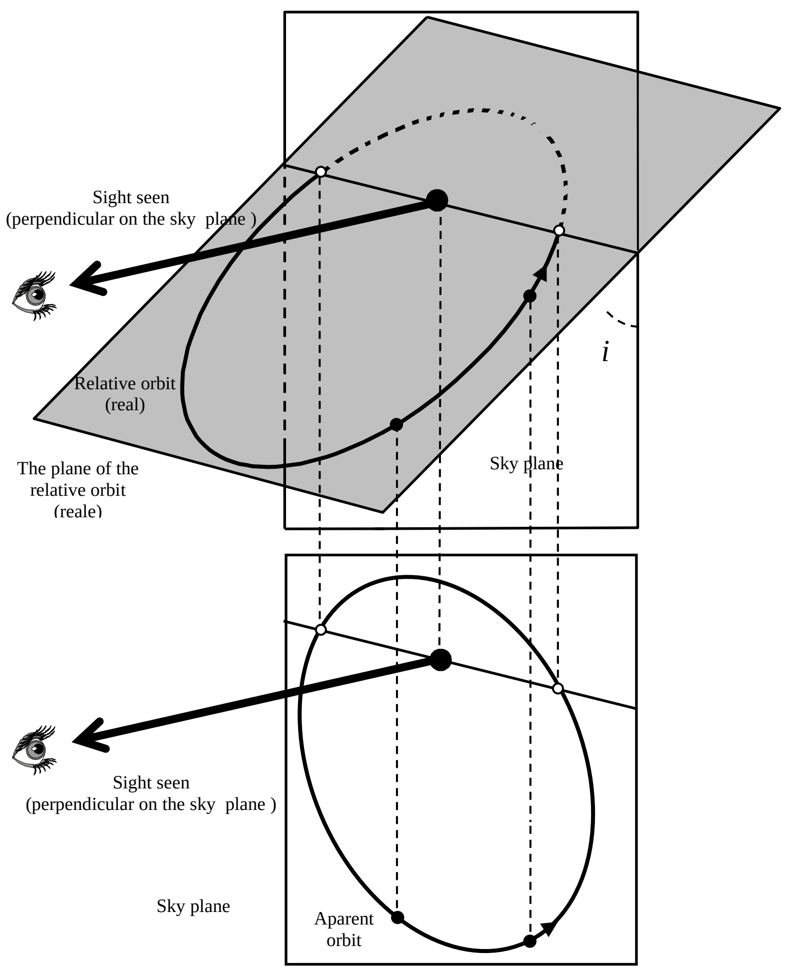
$G = 6{,}67 \cdot 10^{-11}\ \mathrm{Nm^2 kg^{-2}}$

Problem 2. Marking scheme Black Hole in Milky Way

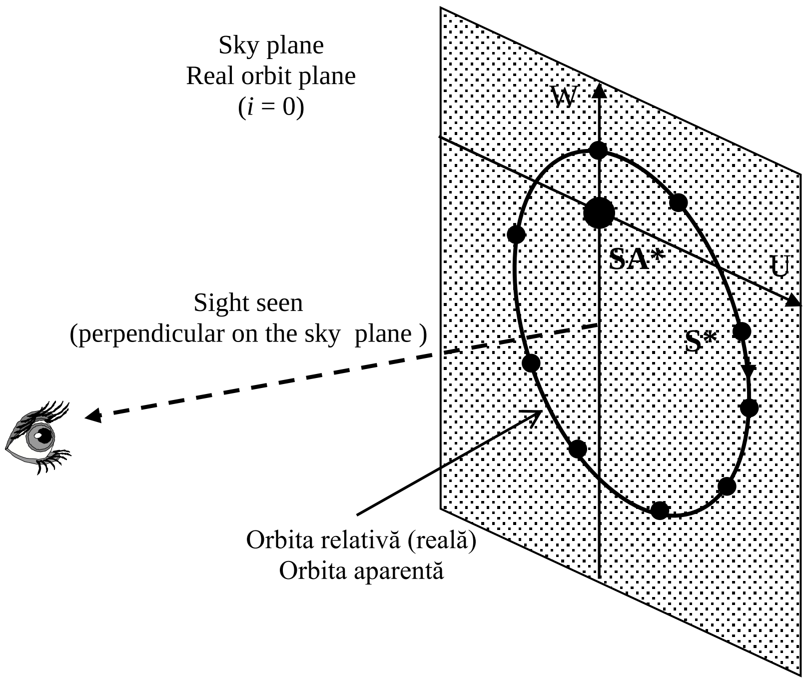
- **a)** **15 points**
- **b)** **10 points**
- **c)** **25 points**
  - I 5 points
  - II 5 points
  - III 5 points
  - IV 5 points
  - V 5 points

Detailed solution

Usually the plane of apparent orbit (sky plane) is different from the plane of the relative orbit due to the angle between the two planes (i = 0).



In the case of the problem the sky plane and the plane of the relative orbit are the same (i = 0), See the figure.



The linear position coordinates (u,w), of the projection of the star S* in the sky plane with the coordinate axes (U, W), having the origin in Sagittarius A*, expressed in light-days are given in the table 1. (for an angular distance $\varphi = 1\,\text{arcsec}$ correspond in sky plane the linear distance $d = 41\,\text{light day}$).

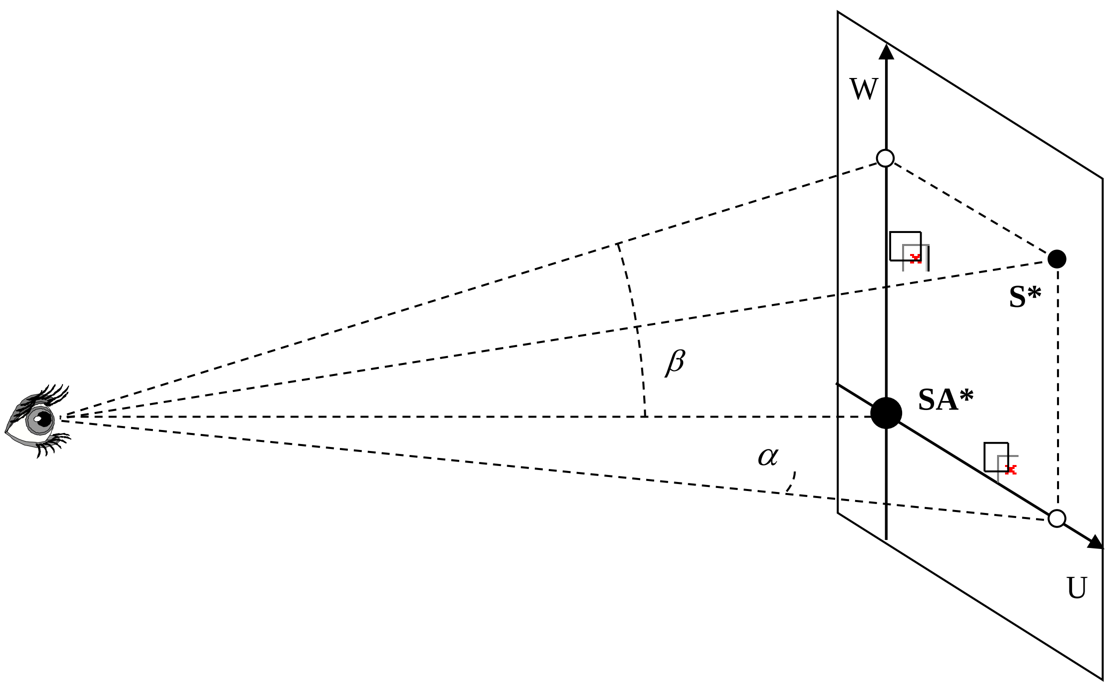


Table 1 The coordinates in light-days

	Date (year)	$\alpha(\text{arcsec})$	$u(\text{ld})$	$\beta(\text{arcsec})$	$w(\text{ld})$
1	1995.222	0.117	4.797	- 0.166	-6.806
2	1997.526	0.097	3.977	- 0.189	-7.749
3	1998.326	0.087	3.567	- 0.192	-7.872
4	1999.041	0.077	3.157	- 0.193	-7.913
5	2000.414	0.052	2.132	- 0.183	-7.503
6	2001.169	0.036	1.476	- 0.167	-6.847
7	2002.831	- 0.000	0.000	- 0.120	-4,920
8	2003.584	- 0.016	-0.656	- 0.083	-3.403
9	2004.165	- 0.026	-1.066	- 0.041	-1.681
10	2004.585	- 0.017	-0.697	0.008	0.328
11	2004.655	- 0.004	-0.164	0.014	0.574
12	2004.734	0.008	0.328	0.017	0.697
13	2004.839	0.021	0.861	0.012	0.492
14	2004.936	0.037	1.517	0.009	0.369
15	2005.503	0.072	2.952	- 0.024	-0.984
16	2006.041	0.088	3.608	- 0.050	-2.050
17	2007.060	0.108	4.428	- 0.091	-3.731

To represent the image of the relative orbit of star S* in a plane parallel with the sky plane we use a scale of and for conversion we use $x = \alpha S$ and $y = \beta S$, see figure 3. These coordinates of the image of the star S* relative to the origin of the axes – the image of Sagittarius A* are given in the below table 3

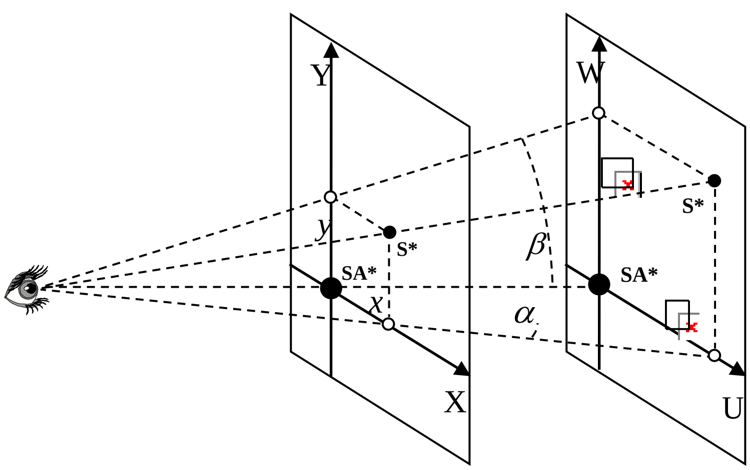


Table 2 The coordinates in scaled plane

	Date (year)	$\alpha(\text{arcsec})$	$x(\text{mm})$	$\beta(\text{arcsec})$	$y(\text{mm})$
1	1995.222	0.117	140.4	- 0.166	-199.2
2	1997.526	0.097	116.4	- 0.189	-226.8
3	1998.326	0.087	104.4	- 0.192	-230.4
4	1999.041	0.077	92.4	- 0.193	-231.6
5	2000.414	0.052	62.4	- 0.183	-219.6
6	2001.169	0.036	43.2	- 0.167	-200.4
7	2002.831	- 0.000	0.0	- 0.120	-144.0
8	2003.584	- 0.016	-20.0	- 0.083	-100.0
9	2004.165	- 0.026	-32.0	- 0.041	-50.0
10	2004.585	- 0.017	-20.4	0.008	9.6
11	2004.655	- 0.004	-4.8	0.014	16.8
12	2004.734	0.008	9.6	0.017	20.4
13	2004.839	0.021	25.2	0.012	14.4
14	2004.936	0.037	44.4	0.009	10.8
15	2005.503	0.072	86.4	- 0.024	-28.8
16	2006.041	0.088	106.0	- 0.050	-60.0
17	2007.060	0.108	130.0	- 0.091	-110.0

The following pictures represents the plot of the values in the table 2, and the measurements which have to be done in order to find out the required data.

Numeric processing

a) From the measurements done on paper system (X, Y):

$$a_{maesured} \approx 135\,\text{mm};\ b_{mesured} \approx 66\,\text{mm};\ d_{\min} = 17\,\text{mm},$$

Corresponding to the numeric scale:

$$a_{mesured} = \varphi_a \cdot S;\ b_{mesured} = \varphi_{b,,} \cdot S;\ d_{\min^{mes}} = \varphi_{\min} \cdot S,$$

$$\varphi_a = 0.1125\,\text{arcsec};$$

$$\varphi_b = 0.055\,\text{arcsec};$$

$$\varphi_{d\min} = 0{,}01416\,\text{arcsec};$$

$$1\,\text{arcsec} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ 41\,\text{light day};$$

$$\varphi_a = 0.1125\,\text{arcsec}\ \ldots\ldots\ldots\ldots\ldots.\ a;$$

$$\varphi_b = 0.055\,\text{arcsec}\ \ldots\ldots\ldots\ldots..\ldots b;$$

$$\varphi_{d\min} = 0{,}014666\,\text{arcsec} \ldots\ldots\ldots\ldots\ldots\ldots..\ r_{\min};$$

$$a = \frac{0.1125\,\text{arcsec} \cdot 41\,\text{light days}}{1\,\text{arcsec}};\ \ a = 4.6125\,\text{light days},$$

$$b = \frac{0.055\,\text{arcsec} \cdot 41\,\text{light days}}{1\,\text{arcsec}};\ b = 2.255\ \text{light days},$$

$$r_{\min} = \frac{0{,}0146\ \text{arcsec} \cdot 41\ \text{light days}}{1\ \text{arcsec}} = 0{,}5986\ \text{light days};$$

$$c = \sqrt{a^2 - b^2} \approx 4{,}0236\ \text{light days};$$

$$r_{\min} = a - c = 0{,}5889\ \text{light days};\ r_{\max} = a + c = 8{,}6361\ \text{light days};$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} \approx 0.87,$$

b) $S_{\text{elipsa}} = 28560\ \text{mm}^2$; See Anex

$$T \ldots\ldots\ldots\ldots\ldots\ldots\ldots S_{\text{elipsa}};$$

$$\Delta t \ldots\ldots\ldots\ldots\ldots \Delta S;$$

$$T = \frac{S_{\text{elipsa}}}{\Delta S} \cdot \Delta t;$$

Displacement $1 \to 2$

$$\Delta S_{12} = \frac{252\ \text{mm x } 34\ \text{mm}}{2} + 86\ \text{mm}^2 \approx 4370\ \text{mm}^2;$$

$$t_1 = 1995.222\ \text{years};\ t_2 = 1997.526\ \text{years};$$

$$\Delta t_{12} = t_2 - t_1 = 2.304\ \text{years};$$

$$T \ \ldots\ldots\ldots\ldots\ldots\ldots\ 28560\ \text{mm}^2;$$

$$2.304\ \text{years} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 4370\ \text{mm}^2;$$

$$T = 15.057\ \text{years};$$

Displacement $2 \to 3$

$$\Delta S_{23} = \frac{252\ \text{mm x } 12\ \text{mm}}{2} + 20\ \text{mm}^2 \approx 1532\ \text{mm}^2;$$

$$t_2 = 1997.526\ \text{years};\ t_3 = 1998.326\ \text{years};$$

$$\Delta t_{23} = t_3 - t_2 = 0.800\ \text{years};$$

$$T \ \ldots\ldots\ldots\ldots\ldots\ldots\ 28560\ \text{mm}^2;$$

$$0.800\ \text{years} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 1532\ \text{mm}^2;$$

$$T = 14.913\ \text{years};$$

Displacement $3 \to 4$

$$\Delta S_{34} = \frac{250\ \text{mm x } 11\ \text{mm}}{2} + 26\ \text{mm}^2 \approx 1400\ \text{mm}^2;$$

$$t_3 = 1998.326\ \text{years};\ t_4 = 1999.041\ \text{years};$$

$$\Delta t_{34} = t_4 - t_3 = 0.715\ \text{years};$$

$$T \ \ldots\ldots\ldots\ldots\ldots\ldots\ 28560\ \text{mm}^2;$$

$$0.715\ \text{years} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 1400\ \text{mm}^2;$$

$$T = 14.586 \text{ years};$$

Displacement $4 \to 5$

$$\Delta S_{45} = \frac{244 \text{ mm x } 22 \text{ mm}}{2} + 30 \text{ mm}^2 = 2714 \text{ mm}^2;$$

$$t_4 = 1999.041 \text{ years}; \ t_5 = 2000.414 \text{ years};$$

$$\Delta t_{45} = t_5 - t_4 = 1.373 \text{ years};$$

$$T \ \ldots\ldots\ldots\ldots\ldots\ldots\ \ 28560 \text{ mm}^2;$$

$$1.373 \text{ years} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ \ 2714 \text{ mm}^2;$$

$$T = 14.448 \text{ years};$$

Displacement $5 \to 6$

$$\Delta S_{56} = \frac{220 \text{ mm x } 13 \text{ mm}}{2} + 5 \text{ mm}^2 = 1435 \text{ mm}^2;$$

$$t_5 = 2000.414 \text{ years}; \ t_6 = 2001.169 \text{ years};$$

$$\Delta t_{56} = 0.755 \text{ years};$$

$$T \ \ldots\ldots\ldots\ldots\ldots\ldots\ \ 28560 \text{ mm}^2;$$

$$0.755 \text{ years} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ \ 1435 \text{ mm}^2;$$

$$T = 15.026 \text{ years};$$

Displacement $6 \to 7$

$$\Delta S_{67} \approx \frac{203 \text{ mm x } 30 \text{ mm}}{2} + 120 \text{ mm}^2 \approx 3165 \text{ mm}^2;$$

$$t_6 = 2001.169 \text{ years}; \ \ t_7 = 2002.831 \text{ years};$$

$$\Delta t_{67} = 1.662 \text{ years};$$

$$T \ \ldots\ldots\ldots\ldots\ldots\ldots\ \ 28560 \text{ mm}^2;$$

$$1.662 \text{ years} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ \ 3165 \text{ mm}^2;$$

$$T = 14.997 \text{ years};$$

Displacement $7 \to 8$

$$\Delta S_{78} \approx \frac{143 \text{ mm x } 20 \text{ mm}}{2} + 5 \text{ mm}^2 \approx 1435 \text{ mm}^2;$$

$$t_7 = 2002.831 \text{ years}; \ t_8 = 2003.584 \text{ years};$$

$$\Delta t_{78} = 0.753 \text{ years};$$

$$T \ \ldots\ldots\ldots\ldots\ldots\ldots\ \ 28560 \text{ mm}^2;$$

$$0.753 \text{ years} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ \ 1435 \text{ mm}^2;$$

$$T = 14.986 \text{ years};$$

Displacement $8 \to 9$

$$\Delta S_{89} \approx \frac{100 \text{ mm x } 22 \text{ mm}}{2} + 7 \text{ mm}^2 \approx 1107 \text{ mm}^2;$$

$$t_8 = 2003.584 \text{ years}; \ t_9 = 2004.165 \text{ years};$$

$$\Delta t_{89} = 0.581\,\text{years};$$

$$T \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 28560\,\text{mm}^2;$$

$$0.581\,\text{years} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 1107\,\text{mm}^2;$$

$$T = 14.989\,\text{years};$$

Displacement $9 \to 10$

$$\Delta S_{9;10} \approx \frac{60\,\text{mm x } 20\,\text{mm}}{2} + 200\,\text{mm}^2 \approx 800\,\text{mm}^2;$$

$$t_9 = 2004.165\,\text{years};\; t_{10} = 2004.585\,\text{years};$$

$$\Delta t_{9,10} \approx 0.420\,\text{years};$$

$$T \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 28560\,\text{mm}^2;$$

$$0.420\,\text{years} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 800\,\text{mm}^2;$$

$$T = 14.994\,\text{years};$$

Displacement $10 \to 11$

$$\Delta S_{10;11} \approx \frac{20\,\text{mm x } 13\,\text{mm}}{2} + 5\,\text{mm}^2 \approx 135\,\text{mm}^2;$$

$$t_{10} = 2004.585\,\text{years};\; t_{11} = 2004.655\,\text{years};$$

$$\Delta t_{10;11} \approx 0.070\,\text{years};$$

$$T \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 28560\,\text{mm}^2;$$

$$0{,}070\,\text{years} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 135\,\text{mm}^2;$$

$$T = 14.808\,\text{years};$$

Displacement $11 \to 12$

$$\Delta S_{11;12} = \frac{22\,\text{mm x } 13\,\text{mm}}{2} + 4\,\text{mm}^2 = 147\,\text{mm}^2;$$

$$t_{11} = 2004.655\,\text{years};\; t_{12} = 2004.734\,\text{years};$$

$$\Delta t_{11;12} \approx 0.077\,\text{years};$$

$$T \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 28560\,\text{mm}^2;$$

$$0{,}077\,\text{years}; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 147\,\text{mm}^2;$$

$$T = 14.960\,\text{years};$$

Displacement $12 \to 13$

$$\Delta S_{12;13} = \frac{30\,\text{mm x } 13\,\text{mm}}{2} + 4\,\text{mm}^2 = 200\,\text{mm}^2;$$

$$t_{12} = 2004.734\,\text{years};\; t_{13} = 2004.839\,\text{years};$$

$$\Delta t_{12;13} \approx 0.105\,\text{years};$$

$$T \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 28560\,\text{mm}^2;$$

$$0{,}105\,\text{years} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 200\,\text{mm}^2;$$

$$T = 14.994\,\text{years};$$

Displacement $13 \to 14$

$$\Delta S_{13;14} = \frac{41\,\text{mm x }9\,\text{mm}}{2} \approx 185\,\text{mm}^2;$$

$$t_{13} = 2004.839\text{ years; } t_{14} = 2004.936\text{ years;}$$

$$\Delta t_{13;14} \approx 0.097\text{ years;}$$

$$T \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 28560\,\text{mm}^2;$$

$$0.097\text{ years}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 185\,\text{mm}^2;$$

$$T = 14.974\text{ years;}$$

Displacement $14 \to 15$

$$\Delta S_{14;15} = \frac{90\,\text{mm x }22\,\text{mm}}{2} + 90\,\text{mm}^2 = 1080\,\text{mm}^2;$$

$$t_{14} = 2004.936\text{ years; } t_{15} = 2005.503\text{ years;}$$

$$\Delta t_{14;15} \approx 0.567\text{ years;}$$

$$T \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 28560\,\text{mm}^2;$$

$$0.567\text{ years}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 1080\,\text{mm}^2;$$

$$T = 14.994\text{ years;}$$

Displacement $15 \to 16$

$$\Delta S_{15;16} = \frac{1}{2}\cdot 120\text{ mm x }17\text{ mm} + 5\,\text{mm}^2 = 1025\,\text{mm}^2;$$

$$t_{15} = 2005.503\text{ years; } t_{16} = 2006.041\text{ years;}$$

$$\Delta t_{15;16} \approx 0.538\text{ years;}$$

$$T \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 28560\,\text{mm}^2;$$

$$0{,}538\text{ years}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 1025\,\text{mm}^2;$$

$$T = 14.990\text{ years;}$$

Displacement $16 \to 17$

$$\Delta S_{16;17} = \frac{168\,\text{mm x }23\,\text{mm}}{2} + 10\,\text{mm}^2 = 1942\,\text{mm}^2;$$

$$t_{16} = 2006.041\text{ years; } t_{17} = 2007.060\text{ years;}$$

$$\Delta t_{16;17} = 1.019\text{ years;}$$

$$T \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 28560\,\text{mm}^2;$$

$$1.019\text{ years}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 1942\,\text{mm}^2;$$

$$T = 14.985\text{ years;}$$

Displacement $17 \to 1$

$$\Delta S_{17;1} = \frac{240\,\text{mm x }43\,\text{m}}{2} + 300\,\text{mm}^2 = 5460\,\text{mm}^2;$$

$$t_{17} = 2007.060\text{ years;}$$

$$\Delta t_{17;1} = 2{,}867\text{ years;}$$

$$T \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; 28560\,\text{mm}^2;$$

$$2{,}867 \text{ years} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 5460\,\text{mm}^2;$$

$$T = 14.996 \text{ years};$$

$$\overline{T} = 14.923 \text{ years};$$

$$T \approx 15 \text{ years}.$$

c) By using the III rd Kepler's law:

$$T^2 = \frac{4\pi^2}{K(M_{\text{SA}*} + M_{\text{S}*})} \cdot a^3;$$

$$M_{\text{SA}*} + M_{\text{S}*} = \frac{4\pi^2 a^3}{KT^2};$$

$$a = 4.6125 \text{ light days}; \; T \approx 15 \text{ years}; \; K = 6{,}67 \cdot 10^{-11} \text{ Nm}^2\text{kg}^{-2};$$

$$1\,\text{ld} \approx 2{,}6 \cdot 10^{13} \text{ m}; 1 \text{ years} \approx 3{,}1 \cdot 10^{7} \text{ s}; \; 4\pi^2 \approx 40;$$

$$(2{,}6)^3 \approx 17{,}6; \; (4{,}6125)^3 \approx 10^2;$$

$$M_{\text{SA}*} + M_{\text{S}*} = \frac{40 \cdot 10^2 \cdot 17{,}6 \cdot 10^{39}}{6{,}67 \cdot 10^{-11} \cdot 225 \cdot 10^{15}} \text{ kg};$$

$$M_{\text{SA}*} + M_{\text{S}*} \approx 4{,}6 \cdot 10^{36} \text{ kg};$$

$$M_{\text{Sun}} \approx 1{,}8 \cdot 10^{30} \text{ kg};$$

$$M_{\text{SA}*} + M_{\text{S}*} \approx 2{,}5 \cdot 10^{6} \cdot M_{\text{Sun}};$$

$$M_{\text{S}*} << M_{\text{SA}*};$$

$$M_{\text{SA}*} \approx 2{,}5 \cdot 10^{6} \cdot M_{\text{Sun}},$$

Problem 3 Extra solar tests

A. Thermodynamic test

A shuttle was launched to investigate the CO_2 atmosphere of two planets P_1 and P_2. When the shuttle was near the each planet, a radio probe was launched toward each of the planet, in vertical direction (the planet radius direction). When the radio -probe movement becomes uniform, it starts sending values of the pressure of the atmosphere. In fig. 3.1 was plotted the atmospheric pressure values (conventional units) of the planet P_1 as function of the time of descending. When the probe touched the surface of the planet P_1 it sends the value of the temperature $T_0 = 700\,\mathrm{K}$ and respectively the value of gravitational acceleration $g_0 = 10\,\mathrm{ms}^{-2}$

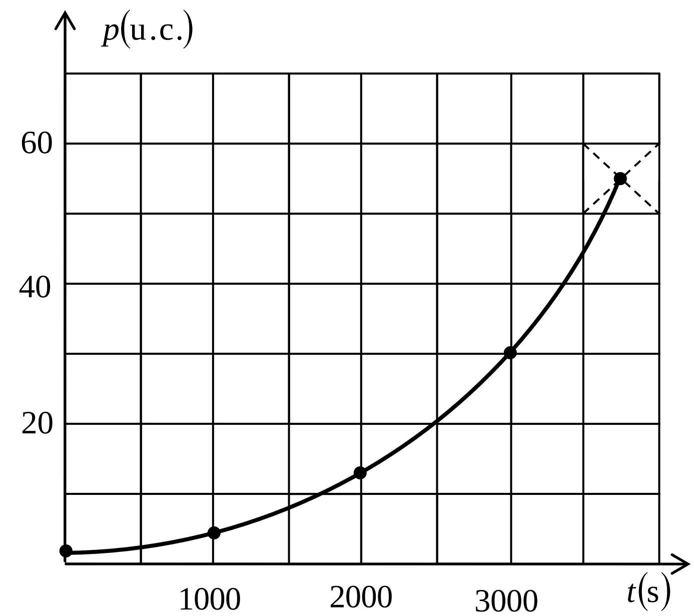


Fig. 3.1.

a) Find out the altitude h_0 from where the radio-probe R_1 starts the uniform descending and thus starts the transmitting information.

b) Find out the temperature of the planet P_1 at the altitude $h = 39{,}6\,\mathrm{km}$. You know: The universal gas constant $R = 8{,}3\,\mathrm{J/molK}$; the molar mass of CO_2, $\mu = 44\,\mathrm{g/mol}$.

c) In fig. 3.2. was plotted the atmospheric pressure values (conventional units) as function of the time of descending in the planet P_2 atmosphere. When the probe touched the surface of the planet P_2, it sends the value of the temperature $T_0 = 750\,\mathrm{K}$ and respectively the value of gravitational acceleration $g_0 = 8\,\mathrm{ms}^{-2}$

Draw the following dependency graphs for $p = f(h)$ and $T = f(h)$ in the CO_2 atmosphere of the planet P_2.

The gravitational acceleration on each planet is constant during uniform descending of the radio –probes.

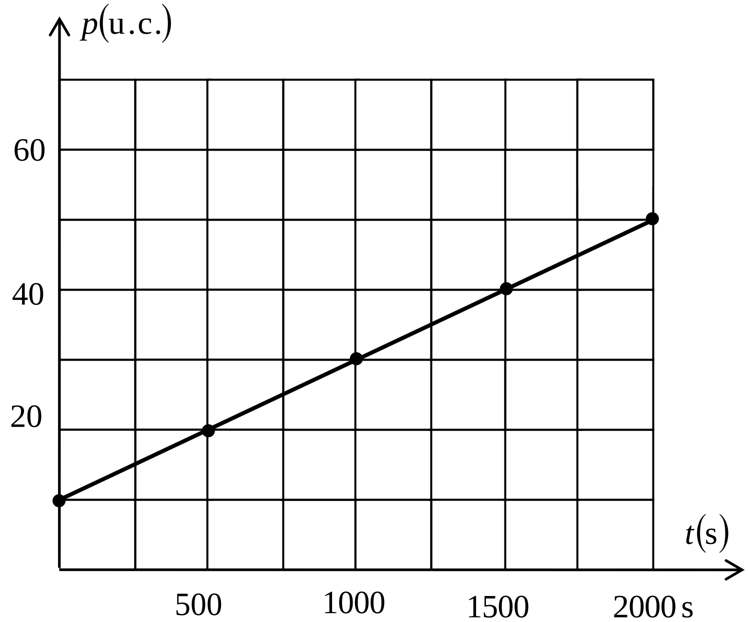


Fig. 3.2.

B. Observer on an extrasolar star

The Sirius star, located in the Canis Majoris constellation is the brightest star which can be observed on the sky with naked eye. What the observer's eye see as a single star is in matter of fact a binary star system.

The luminosity of Sirius is a consequence of two facts: its intrinsic luminosity and its closeness to Earth.

Let's assume that an observer is located on one of the planets of the Sirius system.

In the figure bellow there are represented the Sun, Sirius and the Big Dipper asterism from Ursa Major constellation.

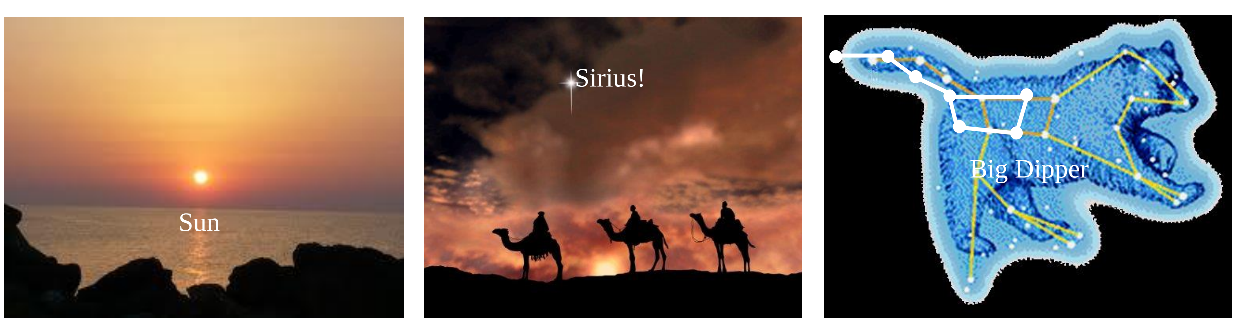


Fig.

Determine:

a) The magnitude of Sun as seen from the observer from the planet of Sirius $m_{Sun,\text{Planet}}$

b) The magnitude of Sirius star system seen by the observer on one of the planets of Sirius. $m_{SY,\text{Planet}}$

c) The luminosity luminozitatea of „Big Dipper”, $L_{\text{Big Dipper}}$;

d) the distance between Big Dipper and Earth, `

e) The geocentric angular distance between Big Dipper and Sirius, $\Delta\theta$;

f) The distance between Big Dipper and the observer from the planet of Sirius $d_{\text{Big Dipper,Planet}}$;

g) The magnitude of Big Dipper as seen by the observer from the planet of Sirius $m_{\text{Big Dipper,Planet}}$

The following data are given:

$d_{\text{Sirius},Earth} = 2{,}6\,\text{pc}$ - the Sirius – Earth distance;

$m_{\text{Sirius},Earth} = -1{,}46^{\text{m}}$ - the apparent magnitude af Sirius measured from Earth;

$L_{\text{Sirius}} = 25 \cdot L_{Sun}$ - the luminosity of Sirius;

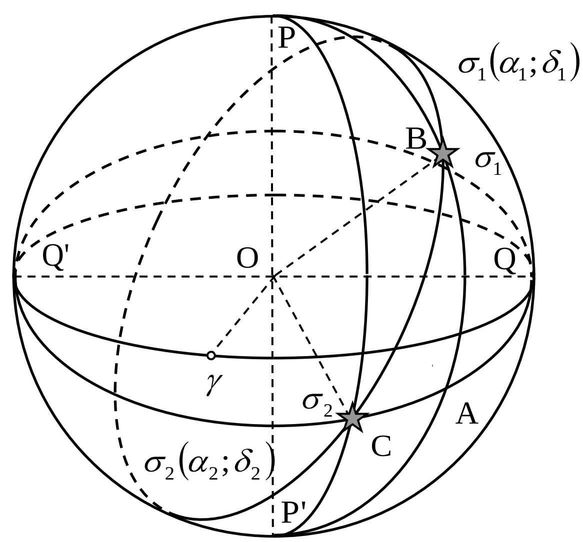
$d_{Sun,Earth} = 1\,\text{AU}$ - distance Sun – Earth ;

$m_{Sun,Earth} = -26{,}78^{\text{m}}$ - the apparent magnitude of Sun as seen from Earth ;

$d_{\text{Sirius,Planet}} = 10\,\text{AU}$ - the distance between Sirius and its planet where the observer is located; $m_{\text{Big Dipper,Pamant}} \approx 2^{\text{m}}$. the apparent magnitude of the Big Dipper asterism for an observer from Earth.

In the table bellow there are information for the stars from Big Dipper asterism regardless to an observer on Earth.

Star number	Name of the star	Apparent magnitude	Distance (light year)
1	Dubhe	1,8	124
2	Merak	2,4	79
3	Phecda	2,4	84
4	Megrez	3,3	58
5	Alioth	1,8	81
6	Mizar	2,1	78
7	Alkaid	1,9	101



The equatorial of Big Dipper (σ_1) asterism and respectively of Sirius (σ_2), located on the geocentric map are :

$\alpha_{\text{Big Dipper}} = \alpha_1 = 13^{\text{h}}23^{\text{min}}55{,}5^{\text{s}}$;

$\delta_{\text{Big Dipper}} = \delta_1 = 54^\circ 55' 31''$; $\alpha_{\text{Sirius}} = \alpha_2 = 6^{\text{h}}45^{\text{min}}$;

$\delta_{\text{Sirius}} = \delta_2 = -16^\circ 43'$.

Problem 3 A. Marking scheme Thermodynamic test **25 points**

a.	5 Points
b.	10 Points
c.	10 points

The pressure and the density of the atmosphere increase with the decrease of the altitude. Th pressure at the surface of the planet is p_{max}.

In figure 1 is represented a thin atmospheric layer with thickness Δh near the surface of the planet P_1 where the atmosphere density can be considered ρ_{max}. Inside this layer the pressure and the temperature are variable, reaching the values p_{max} and T_0 near the surface.

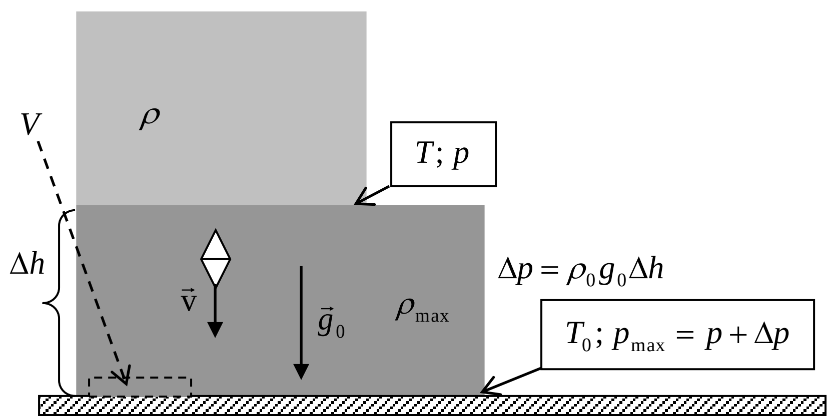


Fig. 1

The following relations can be written :

$$p_{max} = p + \Delta p;$$

$$\Delta p = \rho_{max} g_0 \Delta h,$$

Δp - the variation of the pressure due to the altitude variation Δh, g_0 the gravitational acceleration near the surface of planet P_1;

$$\Delta h = \frac{\Delta p}{\rho_{max} g_0}.$$

The superior layer where the density is constant ρ is represented too. At its base the pressure and temperature hav the values p and respectively T.

For a gas volume inside the near surface layer of the atmosphere,near the surface of the planet for ν mol of de gas according with Mendeleev – Klapeyron law

$$p_{max} V = \nu R T_0 = \frac{m}{\mu} R T_0;$$

$$p_{\max}\mu = \frac{m}{V} RT_0 = \rho_{\max} RT_0;$$

$$\rho_{\max} = \frac{p_{\max}\mu}{RT_0}.$$

Just before touching the surface of the planet P_1 the radio- probe moves uniformly, so the distance Δh is spend with speed $\vec{\mathrm{v}}$, in time Δt:

$$\Delta h = \mathrm{v}\,\Delta t;$$

$$\Delta h = \frac{\Delta p}{\rho_{\max} g_0};\ \rho_{\max} = \frac{p_{\max}\mu}{RT_0};$$

$$\frac{\Delta p}{\rho_{\max} g_0} = \mathrm{v}\,\Delta t;\ \mathrm{v} = \frac{\Delta p}{\rho_{\max} g_0 \Delta t};\ \mathrm{v} = \frac{\Delta p}{\frac{p_{\max}\mu}{RT_0} g_0 \Delta t};$$

$$\mathrm{v} = \frac{RT_0 \Delta p}{g_0 \mu p_{\max} \Delta t}, (1)$$

Where accordingly to fig .2 in the point $\mathrm{F}\,(\tau;\, p_{\max})$, the trigonometric tangent of α is:

$$\tan\alpha = \frac{\Delta p}{\Delta t}.$$

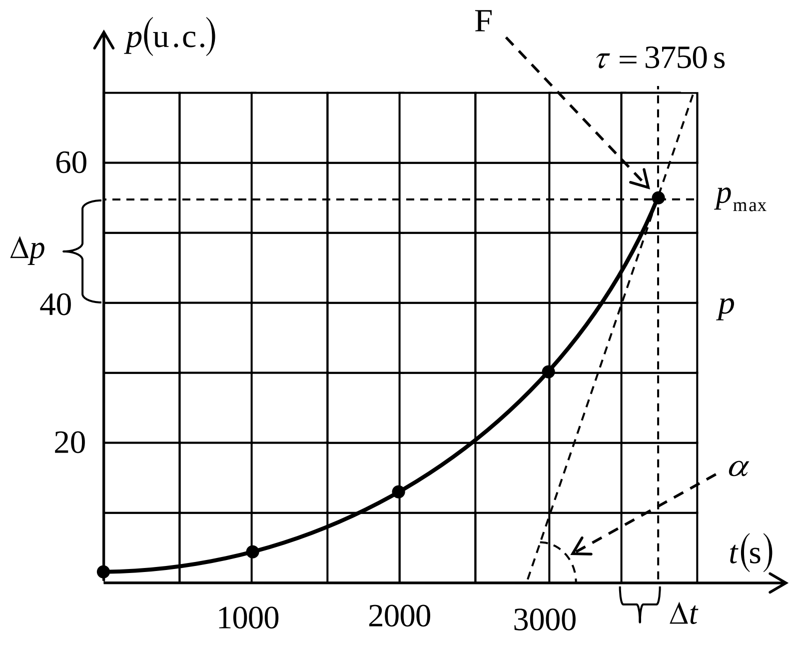


Fig. 2

The rapport :

$$\frac{\Delta p}{p_{\max}\Delta t},$$

Can be determined in the graph in the point F :

$$\mathrm{v}=\frac{\Delta p}{p_{\max}\Delta t}\frac{RT_0}{g_0\mu};$$

$\dfrac{\Delta p}{p_{\max}\Delta t}$	$\dfrac{15\,\mathrm{u.c.}}{55\,\mathrm{u.c.}\,250\,\mathrm{s}}=\dfrac{3}{2750}\dfrac{1}{\mathrm{s}}$	$\dfrac{45\,\mathrm{u.c.}}{55\,\mathrm{u.c.}\,750\,\mathrm{s}}=\dfrac{3}{2750}\dfrac{1}{\mathrm{s}}$

$$\mathrm{v}=\frac{3}{2750}\frac{1}{\mathrm{s}}\frac{8{,}3\dfrac{\mathrm{J}}{\mathrm{molK}}700\,\mathrm{K}}{10\dfrac{\mathrm{m}}{\mathrm{s}^2}\,44\dfrac{10^{-3}\ \mathrm{kg}}{\mathrm{mol}}}=14{,}4\,\frac{\mathrm{m}}{\mathrm{s}};$$

$$h_0=\mathrm{v}\tau;\ \tau=3.750\ \ \mathrm{s};$$

$$h_0=54.000\,\mathrm{m}=54\,\mathrm{km}.$$

b) Descending from the initial altitude $h_0=54\,\mathrm{km}$, with speed $\mathrm{v}=14{,}4\,\dfrac{\mathrm{m}}{\mathrm{s}}$, the radio-probe , after 1000 s arrives at the altitude $h=39{,}6\,\mathrm{km}$

$$t=\frac{h_0-h}{\mathrm{v}}=\frac{14400\,\mathrm{m}}{14{,}4\,\dfrac{\mathrm{m}}{\mathrm{s}}}=1000\,\mathrm{s}.$$

On Graph at altitude $h=39{,}6\,\mathrm{km}$, the atmospheric pressure is $p_h=4\,\mathrm{u.c.}$,- see point C on grah.

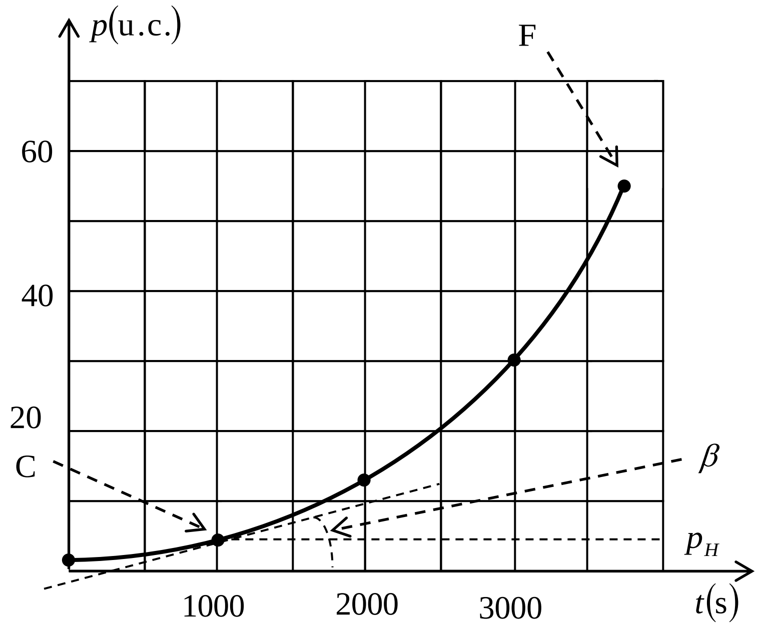


Fig. 3

In this point

$$\tan\beta = \left(\frac{\Delta p}{\Delta t}\right)_C;$$

$$\left(\frac{\Delta p}{p\Delta t}\right)_C = \frac{1}{p_h}\left(\frac{\Delta p}{\Delta t}\right)_C = \frac{1}{4\,\text{u.c.}}\frac{3\,\text{u.c.}}{500\,\text{s}} = \frac{3}{2000}\frac{1}{\text{s}};$$

$$\left(\frac{p\Delta t}{\Delta p}\right)_C = p_h\left(\frac{\Delta t}{\Delta p}\right)_C = 4\,\text{u.c.}\frac{500\,\text{s}}{3\,\text{u.c.}} = \frac{2000}{3}\text{s};$$

$$\text{v} = \left(\frac{\Delta p}{p\Delta t}\right)_C \frac{RT_h}{g_0\mu};$$

$$\text{v} = \frac{1}{p_h}\left(\frac{\Delta p}{\Delta t}\right)_C \frac{RT_h}{g_0\mu};$$

$$T_h = \frac{\text{v}\, g_0\mu}{\frac{1}{p_h}\left(\frac{\Delta p}{\Delta t}\right)_C R};\; T_h = p_h\left(\frac{\Delta t}{\Delta p}\right)_C \frac{\text{v}\, g_0\mu}{R};$$

$$T_h = \frac{2000}{3}\,\text{s}\,\frac{14{,}4\,\frac{\text{m}}{\text{s}}\,10\,\frac{\text{m}}{\text{s}^2}\,44\,\frac{10^{-3}\,\text{kg}}{\text{mol}}}{8{,}3\,\frac{\text{J}}{\text{molK}}};$$

$$T_h = 508{,}9\,\text{K}.$$

c) By using the same algorithm, corresponding to the points F, D, C, B şi A in graph on figure 4 :

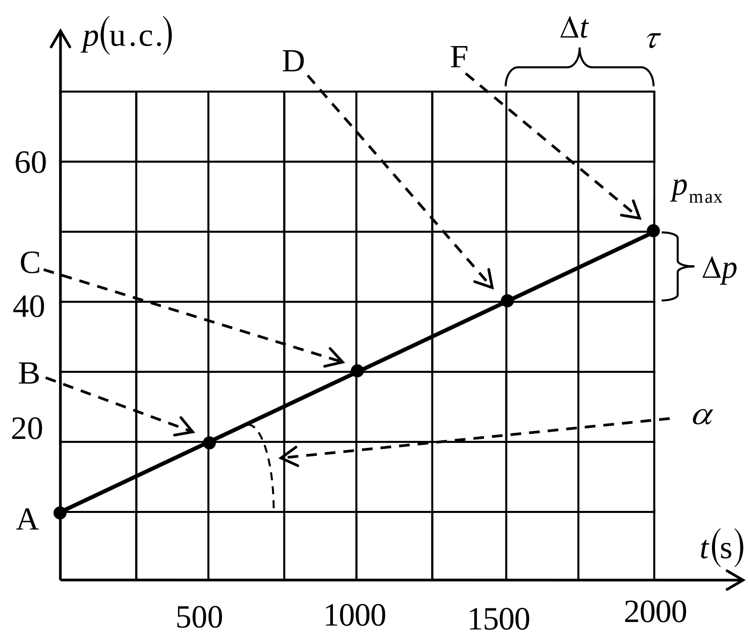


Fig. 4

$$\mathrm{v} = \frac{\Delta p}{p_{\max}\Delta t}\frac{RT_0}{g_0\mu};$$

$$\mathrm{v} = \frac{10\ \mathrm{u.c}}{50\ \mathrm{u.c.}\,500\ \mathrm{s}}\frac{8{,}3\frac{\mathrm{J}}{\mathrm{molK}}750\ \mathrm{K}}{8\frac{\mathrm{m}}{\mathrm{s}^2}44\frac{10^{-3}\ \mathrm{kg}}{\mathrm{mol}}} \approx 7\frac{\mathrm{m}}{\mathrm{s}};$$

$$h_0 = \mathrm{v}\tau = 7\frac{\mathrm{m}}{\mathrm{s}}2000\ \mathrm{s} = 14.000\ \mathrm{m} = 14\ \mathrm{km};$$

$$h_{\mathrm{F}} = 0;$$

$$\mathrm{F}(p = 50\ \mathrm{u.c.}; h = 0\ \mathrm{km}; T = 750\ \mathrm{K});$$

$$T_{\mathrm{D}} = p_{\mathrm{D}}\left(\frac{\Delta t}{\Delta p}\right)_{\mathrm{D}}\frac{\mathrm{v}g_0\mu}{R} = 593{,}7\ \mathrm{K};$$

$$h_{\mathrm{D}} = h_0 - 7\frac{\mathrm{m}}{\mathrm{s}}1500\ \mathrm{s} = 14\ \mathrm{km} - 10.500\ \mathrm{m} = 3{,}5\ \mathrm{km};$$

$$\mathrm{D}(p = 40\ \mathrm{u.c.}; h = 3{,}5\ \mathrm{km}; T = 593{,}7\ \mathrm{K});$$

$$T_{\mathrm{C}} = p_{\mathrm{C}}\left(\frac{\Delta t}{\Delta p}\right)_{\mathrm{C}}\frac{\mathrm{v}g_0\mu}{R} = 445{,}3\ \mathrm{K};$$

$$h_{\mathrm{C}} = h_0 - 7\frac{\mathrm{m}}{\mathrm{s}}1000\ \mathrm{s} = 14\ \mathrm{km} - 7.000\ \mathrm{m} = 7\ \mathrm{km};$$

$$\mathrm{C}(p = 30\ \mathrm{u.c.}; h = 7\ \mathrm{km}; T = 445{,}3\ \mathrm{K});$$

$$T_{\mathrm{B}} = p_{\mathrm{B}}\left(\frac{\Delta t}{\Delta p}\right)_{\mathrm{B}}\frac{\mathrm{v}g_0\mu}{R} = 296{,}8\ \mathrm{K};$$

$$h_{\mathrm{B}} = h_0 - 7\frac{\mathrm{m}}{\mathrm{s}}500\ \mathrm{s} = 14\ \mathrm{km} - 3.500\ \mathrm{m} = 10{,}5\ \mathrm{km};$$

$$\mathrm{B}(p = 20\ \mathrm{u.c.}; h = 10{,}5\ \mathrm{km}; T = 296{,}8\ \mathrm{K});$$

$$T_{\mathrm{A}} = p_{\mathrm{A}}\left(\frac{\Delta t}{\Delta p}\right)_{\mathrm{A}}\frac{\mathrm{v}g_0\mu}{R} = 148{,}4\ \mathrm{K};$$

$$h_{\mathrm{A}} = 14\ \mathrm{km};$$

$$\mathrm{A}(p = 10\ \mathrm{u.c.}; h = 14\ \mathrm{km}; T = 148{,}4\ \mathrm{K}).$$

The data are in table below which allows to plot de graph $p = f(h)$ and $T = f(h)$ for planet P_2 figure 5 and figure 6.

t	0	500 s	1000 s	1500 s	2000 s

p	10 u.c.	20 u.c.	30 u.c.	40 u.c.	50 u.c.
h	14 km	10,5 km	7 km	3,5 km	0
T	148,4 K	296,8 K	445,3 K	593,3 K	750 K

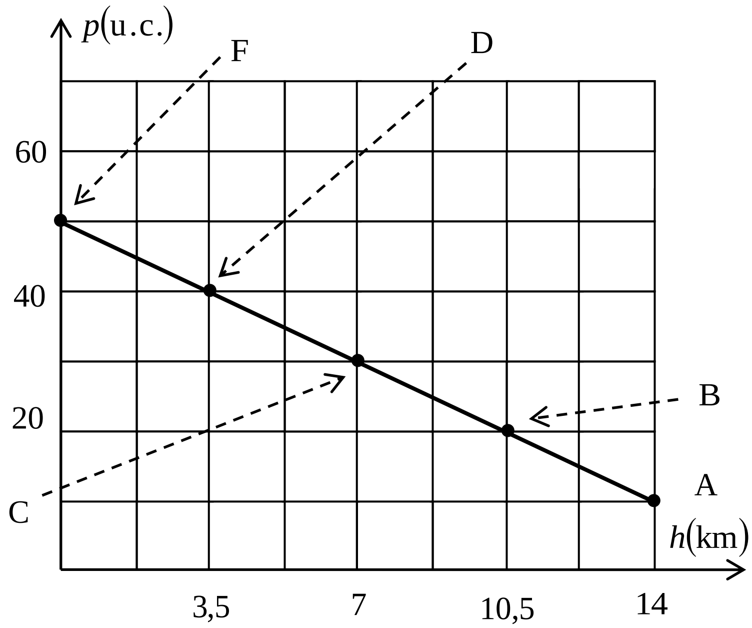


Fig. 5

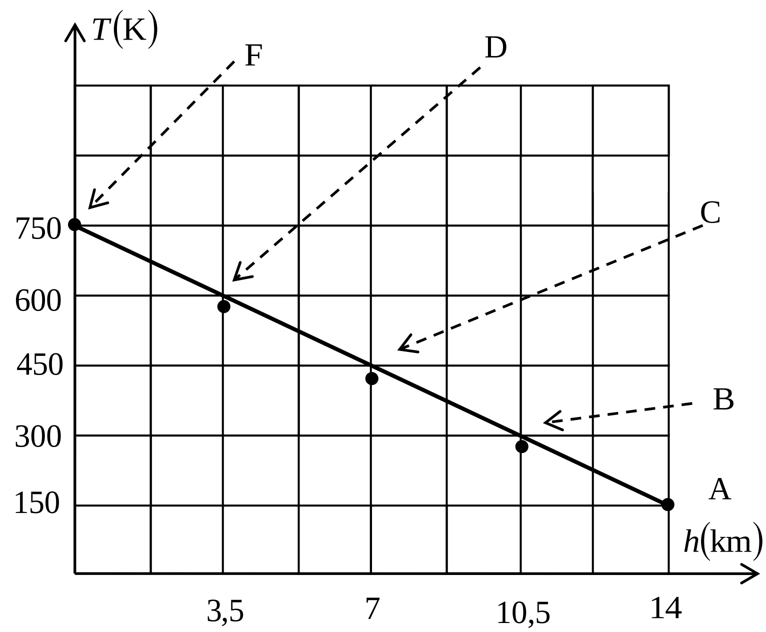


Fig. 6

Problem 3 B. Marking scheme Observer on an extrasolar star **25 points**

a.	3 Points
b.	3 Points
c.	4 Points
d.	4 Points
e.	4 Points
f.	3 Points
g.	4 Points

a) The magnitude of the **SUN as seen from the observer on the Sirrius planet**

$$\log \frac{E_{SUN,EARTH}}{E_{SUN,PLANET}} = -0{,}4\left(m_{SUN,EARTH} - m_{SUN,PLANET}\right),$$

Where: E represents the apparent brightness of the body at the corresponding distance to the observer.

$$\log \frac{\dfrac{L_S}{4\pi d_{S,E}^2}}{\dfrac{L_S}{4\pi d_{S,P}^2}} = -0{,}4\left(m_{S,E} - m_{S,P}\right);$$

$$d_{S,E} = 1\,AU;\ d_{S,P} = 2{,}6\,\text{pc} = 2{,}6\cdot 206265\,AU;$$

$$m_{S,E} = -26{,}78^{m};$$

$$m_{S,P} = 1{,}87^{m},$$

Apparent magnitude of Sun seen from the observer on one of the Sirius planet.

b) **Magnitude of Sirius, seen from the observer from its planet**:

$$\log \frac{E_{\text{Sirius, Planeta}}}{E_{Sun,\text{Planeta}}} = -0{,}4\left(m_{\text{Sirius, Planeta}} - m_{Sun,\text{Planeta}}\right),$$

$$d_{Sun,\text{Planeta}} = 2{,}6\,\text{pc} = 2{,}6\cdot 206265\,A\text{U};$$

$$d_{\text{Sirius, Planeta}} = 10\,A\text{U};$$

$$L_{\text{Sirius}} = 25\cdot L_{SUN};$$

$$m_{Sun,\text{Planeta}} = 1{,}87^{m};$$

$$m_{\text{Sirius, Planeta}} = -25{,}255^{m},$$

c) The luminosity of **Big Dipper**:

$$L_{\text{Big Dipper}} = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7,$$

where $L_1,\ L_2,\ \ldots..,L_7$ are the luminosties of the seven stars of the asterism
:

$$\log\frac{E_{\mathrm{Sun},Earth}}{E_{1,Earth}}=-0{,}4\left(m_{SUN,Earth}-m_{1,Earth}\right),$$

$$\log\frac{\dfrac{L_{\mathrm{Sun}}}{4\pi d^2_{\mathrm{Sun},Earth}}}{\dfrac{L_1}{4\pi d^2_{1,Earth}}}=-0{,}4\left(m_{Sun,Earth}-m_{1,Earth}\right);$$

$$d_{1,Earth}=124\cdot 63240\ A\mathrm{U};$$

$$m_{Sun,Earth}=-26{,}78^{\mathrm{m}};\ m_{1,Earth}=1{,}8^{\mathrm{m}};$$

$$L_1\approx 225\cdot L_{\mathrm{S}},$$

The luminosity of star 1(Dubhe) from the asterism Big Dipper.

Symilarlyfor the rest of stars in the asterism :

$$L_2\approx 50\cdot L_{\mathrm{S}};$$

$$L_3\approx 60\cdot L_{\mathrm{S}};$$

$$L_4\approx 0{,}134\cdot L_{\mathrm{S}};$$

$$L_5\approx 52\cdot L_{\mathrm{S}};$$

$$L_6\approx 4\cdot L_{\mathrm{S}};$$

$$L_7\approx 1{,}37\cdot L_{\mathrm{S}}.$$

The luminosity of the asterism Big Dipper is :

$$L_{\text{Big Dipper}}=L_1+L_2+L_3+L_4+L_5+L_6+L_7;$$

$$L_{\text{Big Dipper}}=(225+50+60+0{,}134+52+4+1{,}37)L_{Sun};$$

$$L_{\text{Big Dipper}}=392{,}5\cdot L_{\mathrm{S}}un.$$

d) **The distance between Big Dipper and Earth** $d_{\text{Big Dipper},Earth}$, :

$$\log\frac{E_{\text{Big Dipper},Earth}}{E_{\mathrm{Soare},Earth}}=-0{,}4\left(m_{\text{Big Dipper},Earth}-m_{\mathrm{Soare},Earth}\right),$$

$$L_{\text{Big Dipper}}=392{,}5\cdot L_{\mathrm{S}};$$

$$d_{\text{Big Dipper},Earth}=573\cdot 10^3\cdot\sqrt{392{,}5}\cdot d_{\mathrm{Soare},Earth};$$

$$d_{\text{Big Dipper},Earth}\approx 11{,}3\cdot 10^6\ AU.$$

e) Geocentric angular distance $\Delta\theta$, **between dintre Big Dipper and Sirius**.

The equatorial coordinates of asterism Big Dipper (σ_1) and Sirius (σ_2) located on the sphere of geocentric celestial sphere are :

$$\alpha_{\text{Big Dipper}}=\alpha_1=13^{\mathrm{h}}23^{\mathrm{min}}55{,}5^{\mathrm{s}};\ \delta_{\text{Big Dipper}}=\delta_1=54^{\circ}55'31'';$$

$$\alpha_{\text{Sirius}} = \alpha_2 = 6^{\text{h}} 45^{\text{min}}; \ \delta_{\text{Sirius}} = \delta_2 = -16^{\circ} 43'.$$

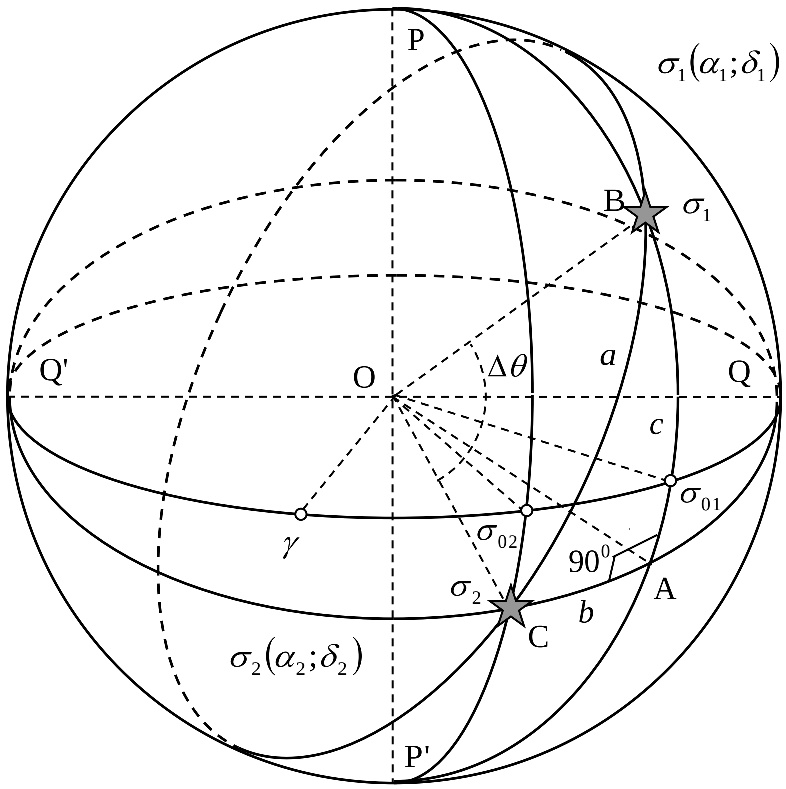


Fig.

In the spherical dreptunghic triangle ABC:

$$<(\sigma_1 \mathrm{O} \sigma_2) = \Delta\theta.$$

Also

$$<(\gamma \mathrm{O} \sigma_{01}) = \alpha_{\text{Big Dipper}} = \alpha_1; <(\sigma_{01} \mathrm{OB}) = \delta_{\text{Big Dipper}} = \delta_1;$$

$$<(\gamma \mathrm{O} \sigma_{02}) = \alpha_{\text{Sirius}} = \alpha_2; <(\sigma_{02} \mathrm{OC}) = \delta_{\text{Sirius}} = \delta_2;$$

$$<(\sigma_{01} \mathrm{O} \sigma_{02}) = <(\mathrm{AOC}) = \alpha_1 - \alpha_2 = \Delta\alpha;$$

$$<(\sigma_1 \mathrm{OA}) = <(\sigma_{01} \mathrm{O} \sigma_1) + <(\sigma_{01} \mathrm{OA}) = \delta_{\text{Big Dipper}} - \delta_{\text{Sirius}} = \Delta\delta;$$

$$<(\sigma_1 \mathrm{O} \sigma_2) = \Delta\theta.$$

Results

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A;$$

$$A = 90^0;$$

$$\cos a = \cos b \cdot \cos c;$$

$$a = \Delta\theta; \ b = \Delta\alpha; \quad c = \Delta\delta;$$

$$\cos\Delta\theta = \cos\Delta\alpha \cdot \cos\Delta\delta;$$

$$\Delta\alpha = \alpha_1 - \alpha_2 = 13^{\text{h}} 23^{\text{min}} 55{,}5^{\text{s}} - 6^{\text{h}} 45^{\text{min}} = 6^{\text{h}} 38^{\text{min}} 55{,}5' \approx 100^{\circ};$$

$$\Delta\delta = \delta_1 - \delta_2 = 54^\circ 55'31'' + 16^\circ 43' = 71^\circ 38'31'' \approx 72{,}4^\circ;$$
$$\cos\Delta\theta = \cos(100^\circ)\cdot\cos(72{,}4^\circ) = (-0{,}173648177)\cdot(0{,}30236989) = -0{,}05250598;$$
$$\Delta\theta \approx 90{,}3^\circ \approx 90^\circ,$$

f) The distance between **Big Dipper and the observer on sirius planet of Sirius**, $d_{\text{Big Dipper,Planeta}}$:

$$d_{\text{Big Dipper,Planeta}} \approx d_{\text{Big Dipper,Sirius}};$$
$$d_{\text{Sirius},Earth} \approx d_{\text{Sirius,Soare}} \approx d_{\text{Soare,Planeta}};$$
$$d_{\text{Sirius, Pamant}} = 2{,}6\ \text{pc};$$
$$d_{\text{Soare,Planeta}} = 2{,}6\ \text{pc} = 2{,}6\cdot 206265\ \text{UA} = d_{Earth};$$
$$d_{\text{Big Dipper},Earth} \approx 11{,}3\cdot 10^6\ AU;$$

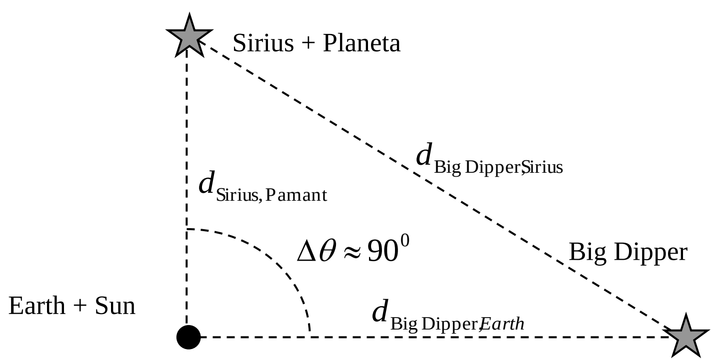


Fig.

$$d_{\text{Big Dipper,Sirius}} = \sqrt{d^2_{\text{Big Dipper,Pamant}} + d^2_{\text{Sirius},Eaarth}} = d_{\text{Big Dipper,Planeta}} \approx 11{,}312\cdot 10^6\ \text{UA}.$$

g) The magnitude of **Big Dipper for an observer from the observer on planet of Sirius** :

$$\log\frac{E_{\text{Big Dipper,Planeta}}}{E_{\text{Big Dipper},Earth}} = -0{,}4\left(m_{\text{Big Dipper,Planeta}} - m_{\text{Big Dipper},Earth}\right),$$
$$m_{\text{Big Dipper},Earth} \approx 2^{\text{m}};\ d_{\text{Big Dipper},Earth} \approx 11{,}3\cdot 10^6\ A\text{U};$$
$$d_{\text{Big Dipper,Sirius}} = d_{\text{Big Dipper,Planeta}} \approx 11{,}312\cdot 10^6\ \text{A}U;$$
$$m_{\text{Big Dipper,Planeta}} = 2^{\text{m}} + 5^{\text{m}}\cdot\log\frac{11{,}312\cdot 10^6\ \text{AU}}{11{,}3\cdot 10^6\ \text{AU}};$$
$$m_{\text{Big Dipper,Planeta}} = 2^{\text{m}} + 5^{\text{m}}\cdot\log\frac{11{,}312}{11{,}3};$$
$$m_{\text{Big Dipper,Planeta}} \approx 2{,}0023^{\text{m}},$$