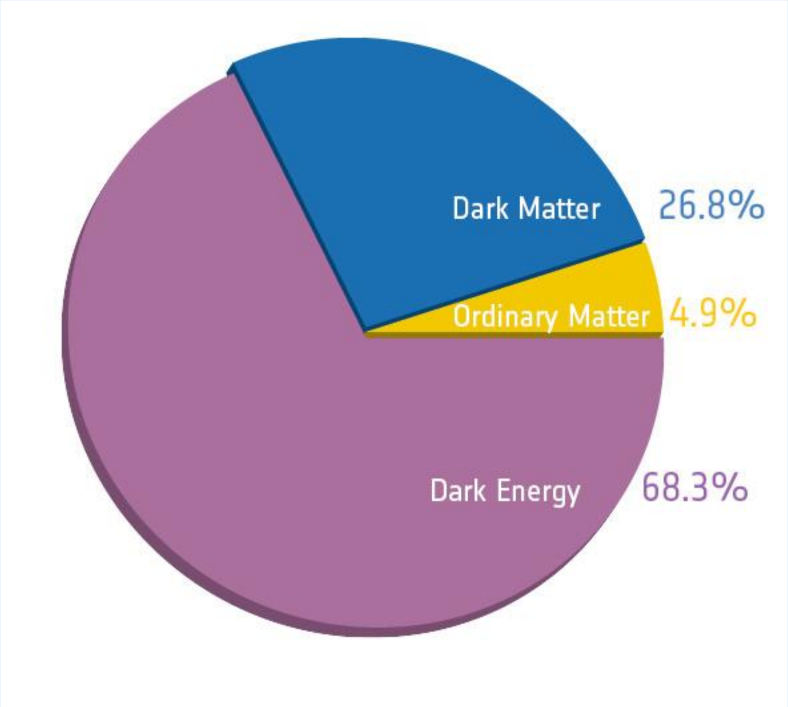
Data Analysis: D-2

“The Search for Dark Matter”

D-2, The Search for Dark Matter

Motivation

- Dark matter
  - Invisible Mass
  - 26.8% of the total mass–energy of the Universe



Planck Collaboration et al. 2013

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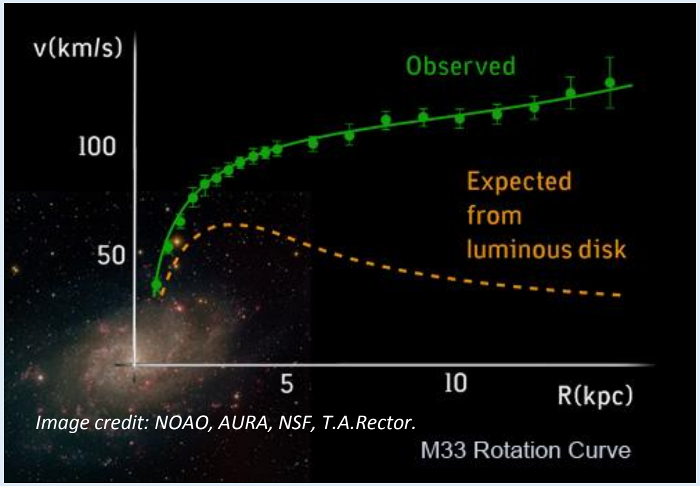
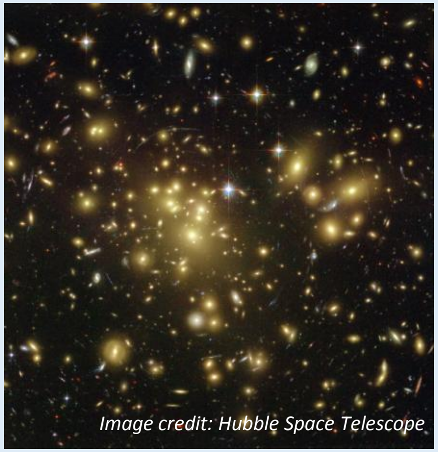
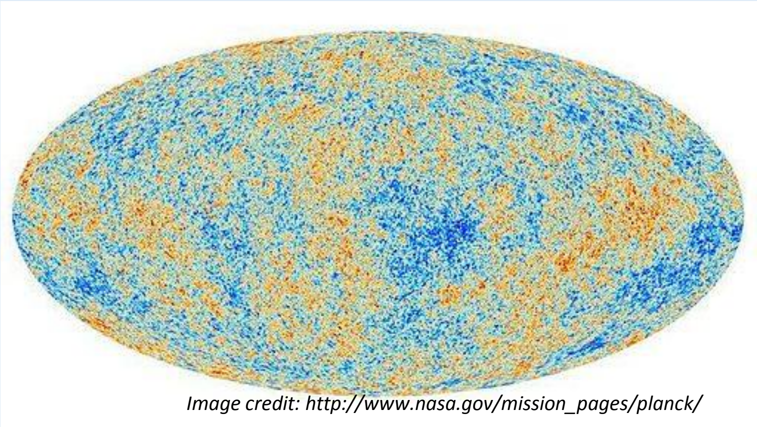


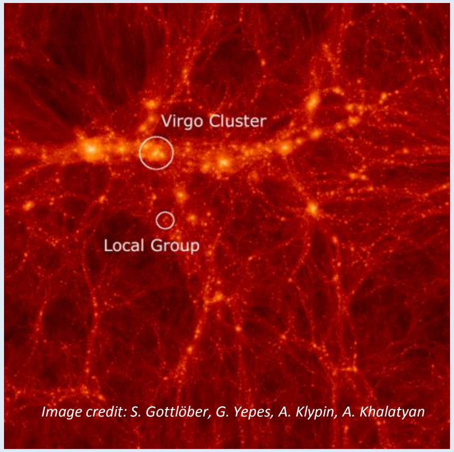
Rotation Curves of Galaxies



Gravitational Lensing



Cosmic Microwave Background



Large Scale Structure

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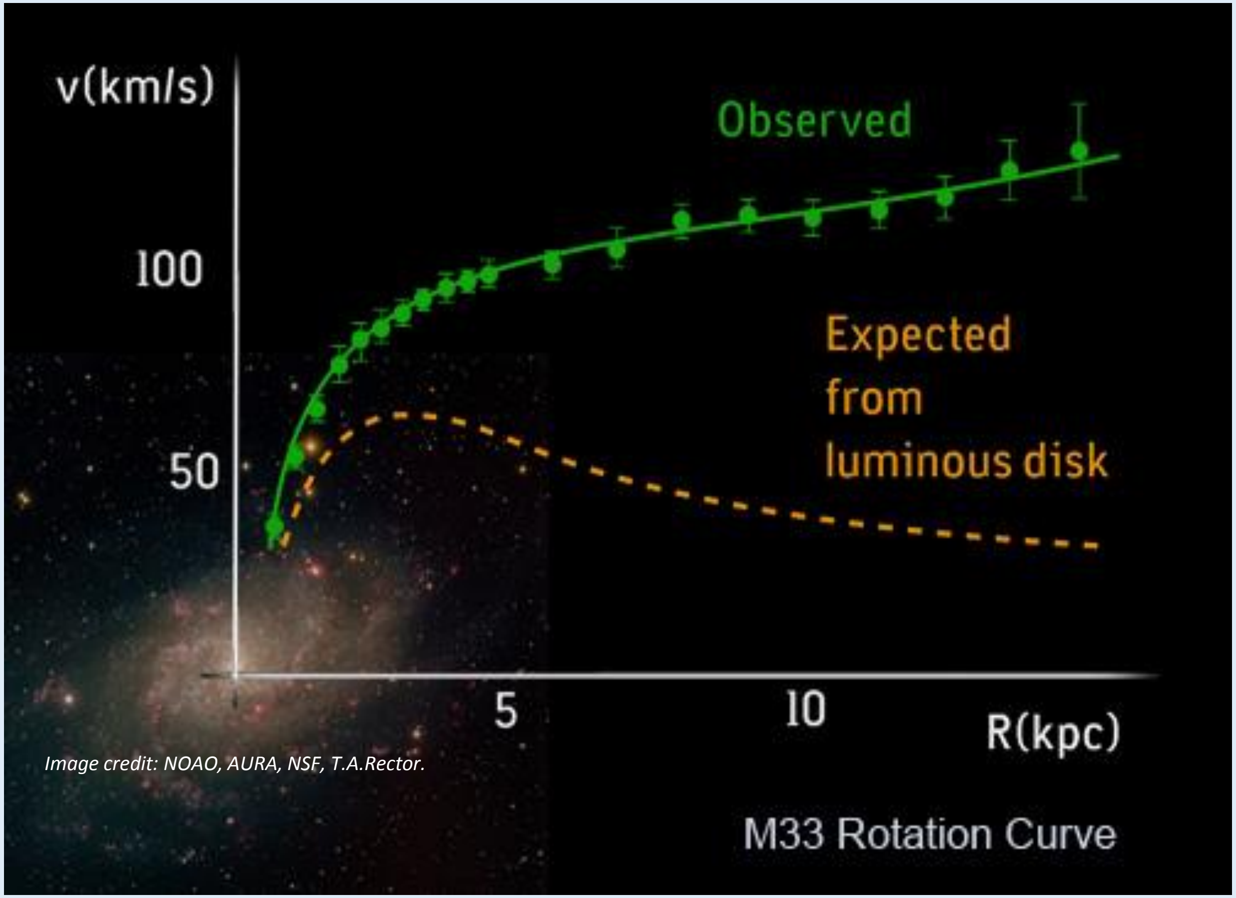


Rotation Curve

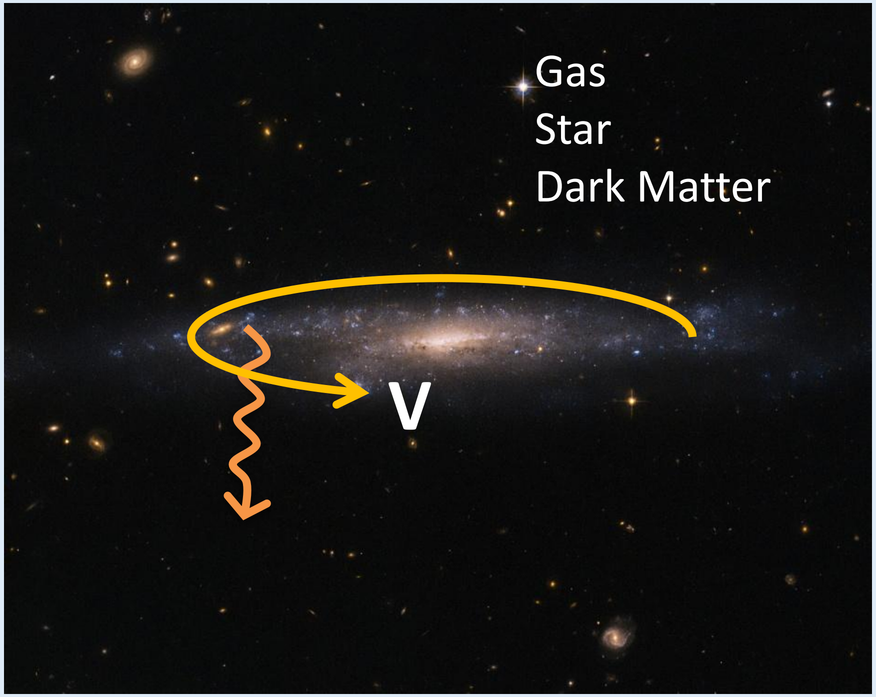
Introduction to the problem

- Dark matter in a low surface brightness galaxy UGC 4325

$$V_{\mathrm{obs}} = zc$$

$$V_{\mathrm{obs}}^2 = V_{\mathrm{gas}}^2 + V_*^2 + V_{\mathrm{DM}}^2$$

$$M_{\mathrm{DM}}(r) = \frac{rV_{\mathrm{DM}}^2}{G}$$



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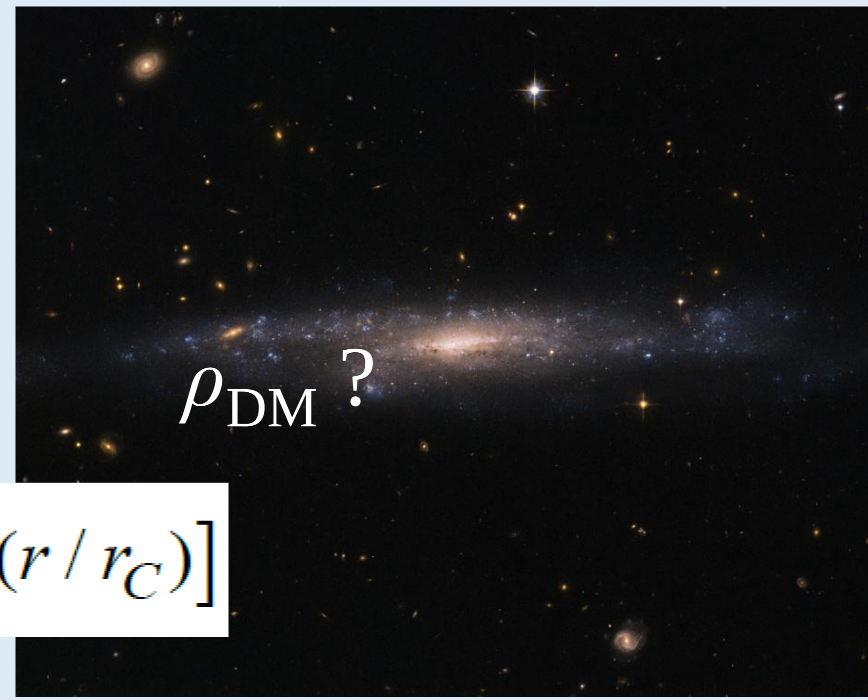
Introduction to the problem

- The density distribution of dark matter

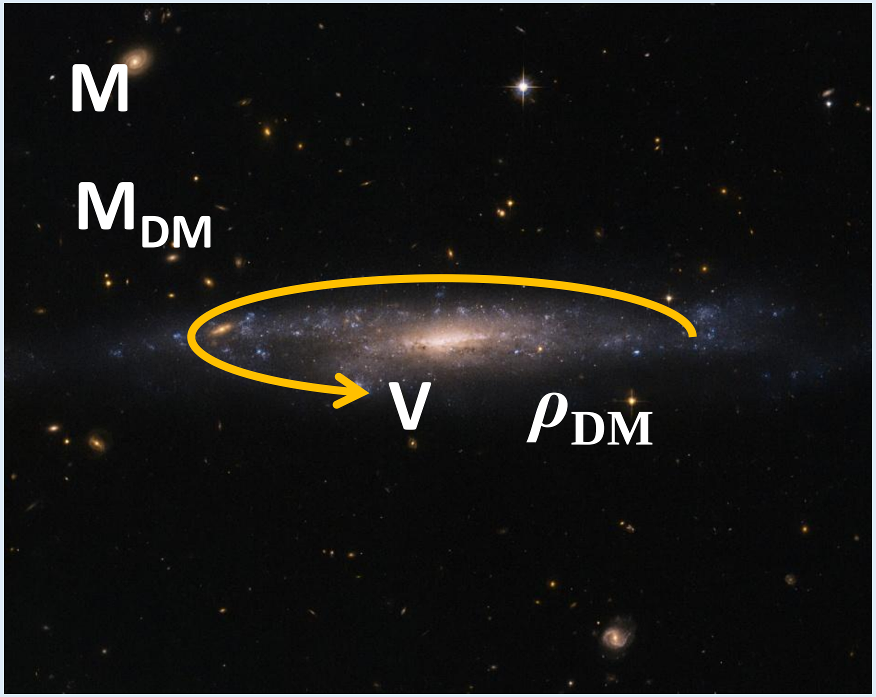
$$\rho_{\mathrm{DM}}(r) = \frac{\rho_0}{1+\left(\dfrac{r}{r_{\mathrm{C}}}\right)^2}$$



$$M_{\mathrm{DM}}(r) = 4\pi\rho_0 r_{\mathrm{C}}^2\left[r - r_C \arctan(r/r_C)\right]$$

Objectives

1. Express the rotation curve of UGC4325
2. Find the mass of the galaxy and the mass of dark matter in the galaxy
3. Predict the density distribution of dark matter in the galaxy.



Part 1 The mass of dark matter and rotation curves of the galaxy

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Provided data

r (kpc)	λ_{obs} (nm)	V_{gas} (km/s)	V_* (km/s)
0.70	656.371	2.87	20.97
1.40	656.431	6.75	32.22
2.09	656.464	14.14	40.91
2.79	656.475	20.18	46.75
3.49	656.478	24.08	50.10
4.89	656.484	28.08	47.94
6.25	656.481	29.25	45.47
7.10	656.481	27.03	47.78
9.03	656.482	25.90	45.32
12.05	656.482	21.03	42.30

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Task D2.1 (21 marks)

- In laboratories on Earth, Hα has an emitted wavelength of 656.281 nm. Compute the observed rotational velocities V_{obs} of the galaxy and the rotational velocities due to the dark matter V_{DM} at distance r in units of km/s.
- For the different values of r given in the table, compute the dynamical mass $M(r)$ and the mass of dark matter $M_{DM}(r)$ in solar masses.

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- Redshift

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}}$$

- Observed rotation velocities

$$V_{\text{obs}} = zc$$

- Rotation velocities due to the dark matter

$$V_{\text{obs}}^2 = V_{\text{gas}}^2 + V_*^2 + V_{\text{DM}}^2$$

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- The mass of the galaxy

$$M(r) = \frac{rV_{\text{obs}}^2}{G}$$

- Mass of dark matter

$$M_{\text{DM}}(r) = \frac{rV_{\text{DM}}^2}{G}$$

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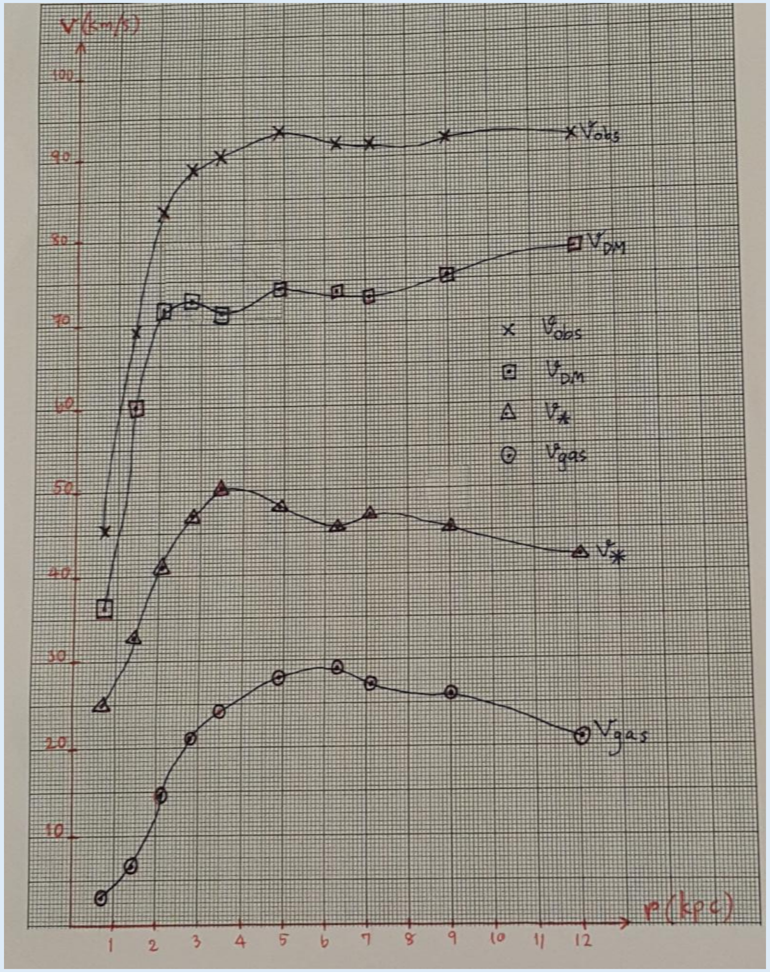
Task D2.2 (16 marks)

- Create rotation curves of the galaxy
- Order the contribution of the different components



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Task D2.3 (7 marks)

- Take a data points at small *r* and large *r* to estimate ρ_0 and r_C .

$$M_{DM}(r) = 4\pi\rho_0 r_C^2 \left[r - r_C \arctan(r / r_C)\right]$$

$$M_{DM}(r) = 4\pi\rho_0 r_C^3 [x - \arctan(x)], \text{ where } x = r/r_C$$

$$M_{DM}(r) \approx 4\pi\rho_0 r_C^3 \left[x - (x - \frac{x^3}{3})\right], \text{ for small } x$$

$$M_{DM}(r) \approx 4\pi\rho_0 r_C^3 \left(\frac{x^3}{3}\right) = \frac{4\pi\rho_0 r^3}{3}$$

$$\rho_0 \approx \frac{2.02\times10^8 M_\odot \times 3}{4\pi(0.7kpc)^3} = 1.42 \times 10^8 M_\odot / kpc^3$$

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$M_{DM}(r) \approx 4\pi\rho_0 r_C^2 \left[r - r_C \frac{\pi}{2}\right],$

Take the last two data points at large r, then we get (for $r \gg r_C$

$\Delta M_{DM}(r) \approx 4\pi\rho_0 r_C^2 [\Delta r]$

$r_C = 1.01\ kpc$

Part 2 Dark matter distribution

?

$$\rho_{DM}(r) = \frac{\rho_0}{1 + \left(\frac{r}{r_C}\right)^2}$$ ✓

$$M_{DM}(r) = 4\pi\rho_0 r_C^2 \left[r - r_C \arctan(r / r_C)\right]$$

✓

Task D2.4 (19 marks)

- Compare Equation (4) to the linear function

$$M_{\mathrm{DM}}(r) = 4\pi\rho_0 r_{\mathrm{C}}^2 \left[r - r_C \arctan(r / r_C) \right]$$

$$y = mx$$

- or

$$\frac{M_{\mathrm{DM}}(r)}{r} = -4\pi\rho_0 r_{\mathrm{C}}^2 \left[\frac{r_C}{r} \arctan(r / r_C) \right] + 4\pi\rho_0 r_{\mathrm{C}}^2$$

$$y = mx + c$$

- The central density ρ_0 can be evaluated from the slope of the best fit

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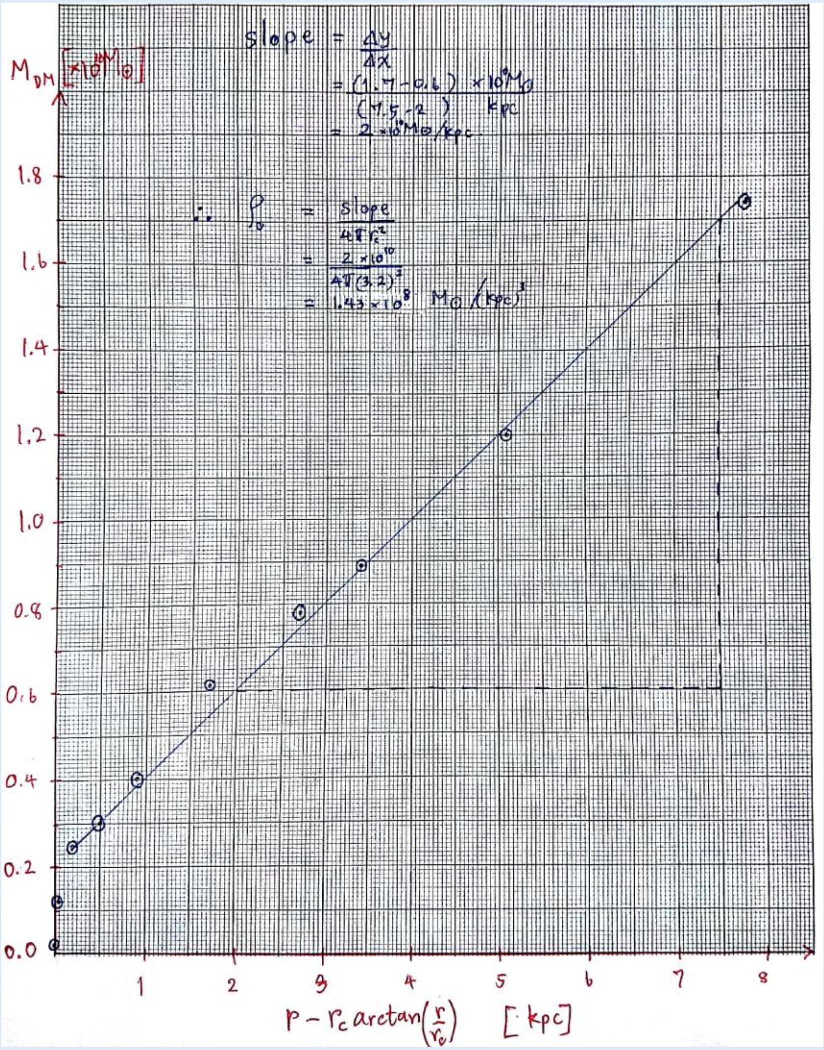
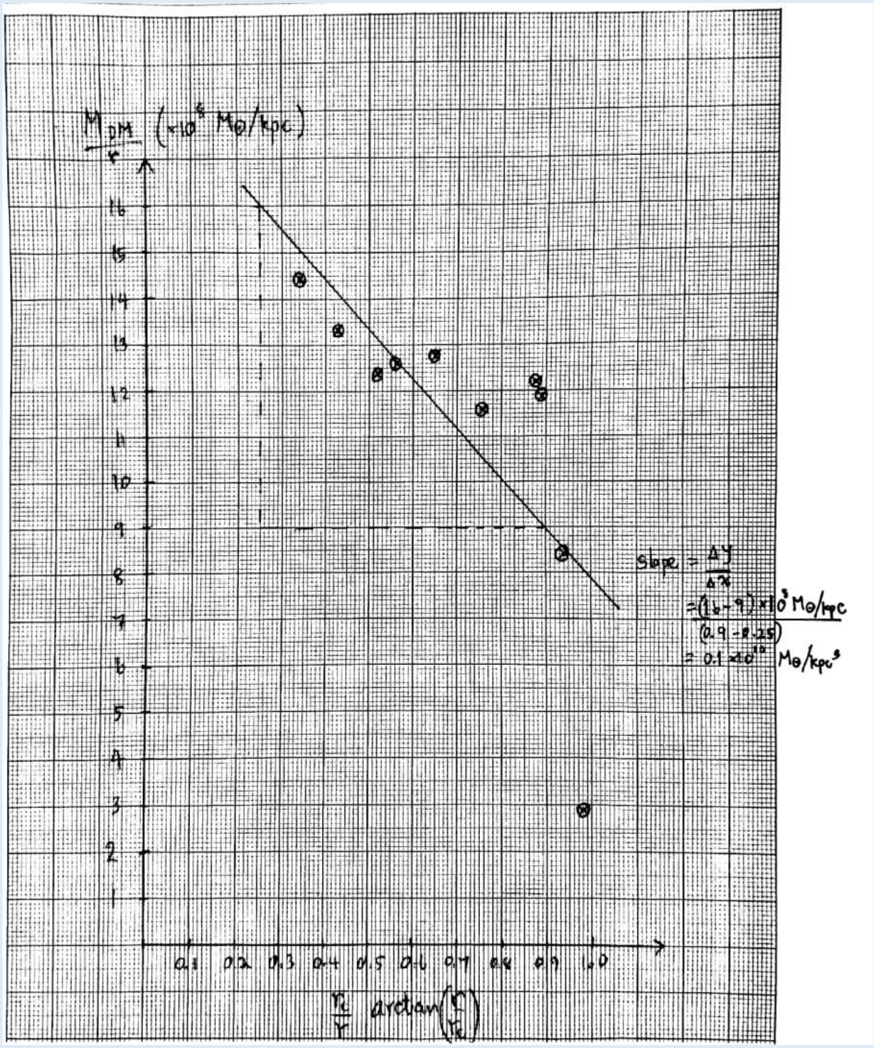
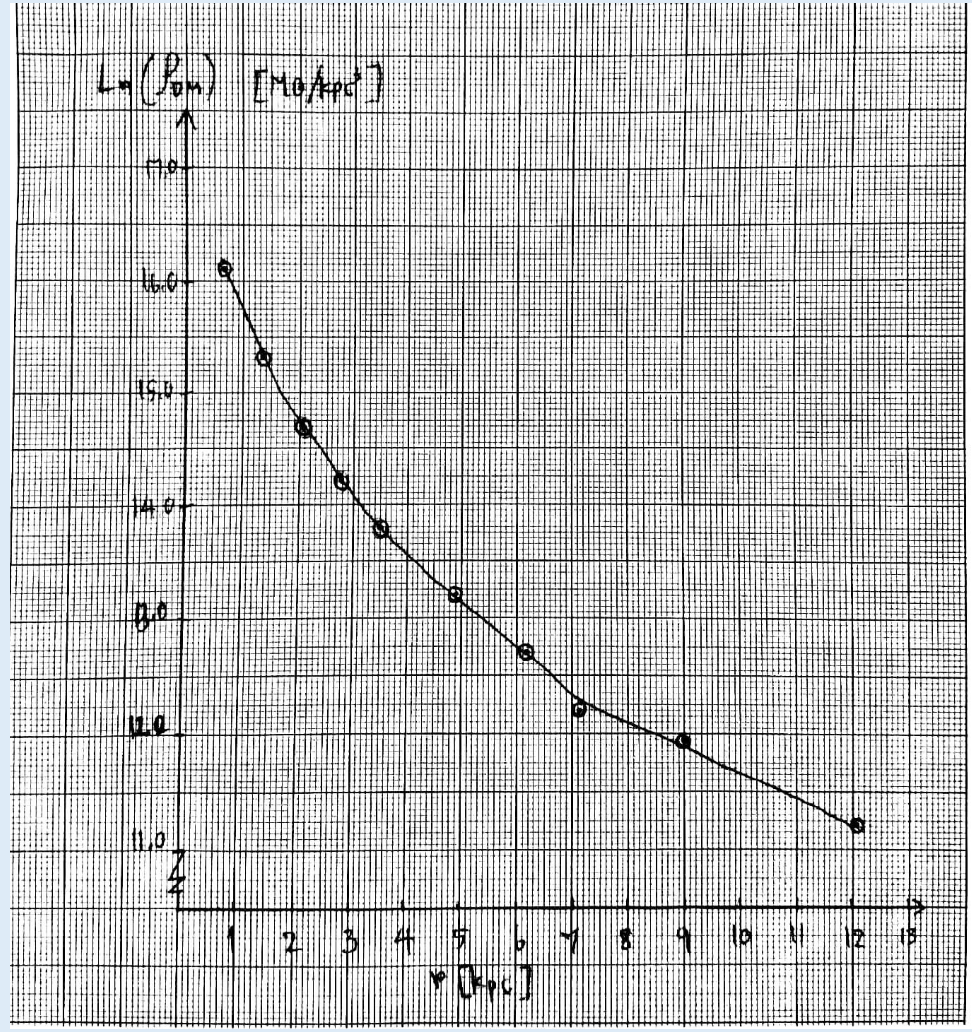




Task D2.5 (12 marks)

- Demonstrate the distribution of the dark matter

$$\rho_{DM}(r) = \frac{\rho_0}{1+\left(\frac{r}{r_C}\right)^2}$$



Knowledge

- Basic Astrophysics:
  - Celestial Mechanics
- Stellar System
  - Galaxies
- Cosmology
  - Elementary Cosmology