Phuket, Thailand 2017

Long Question: T-11

"Mass of the Local Group Galaxy"

T-11, Mass of the Local Group Galaxy

Motivation

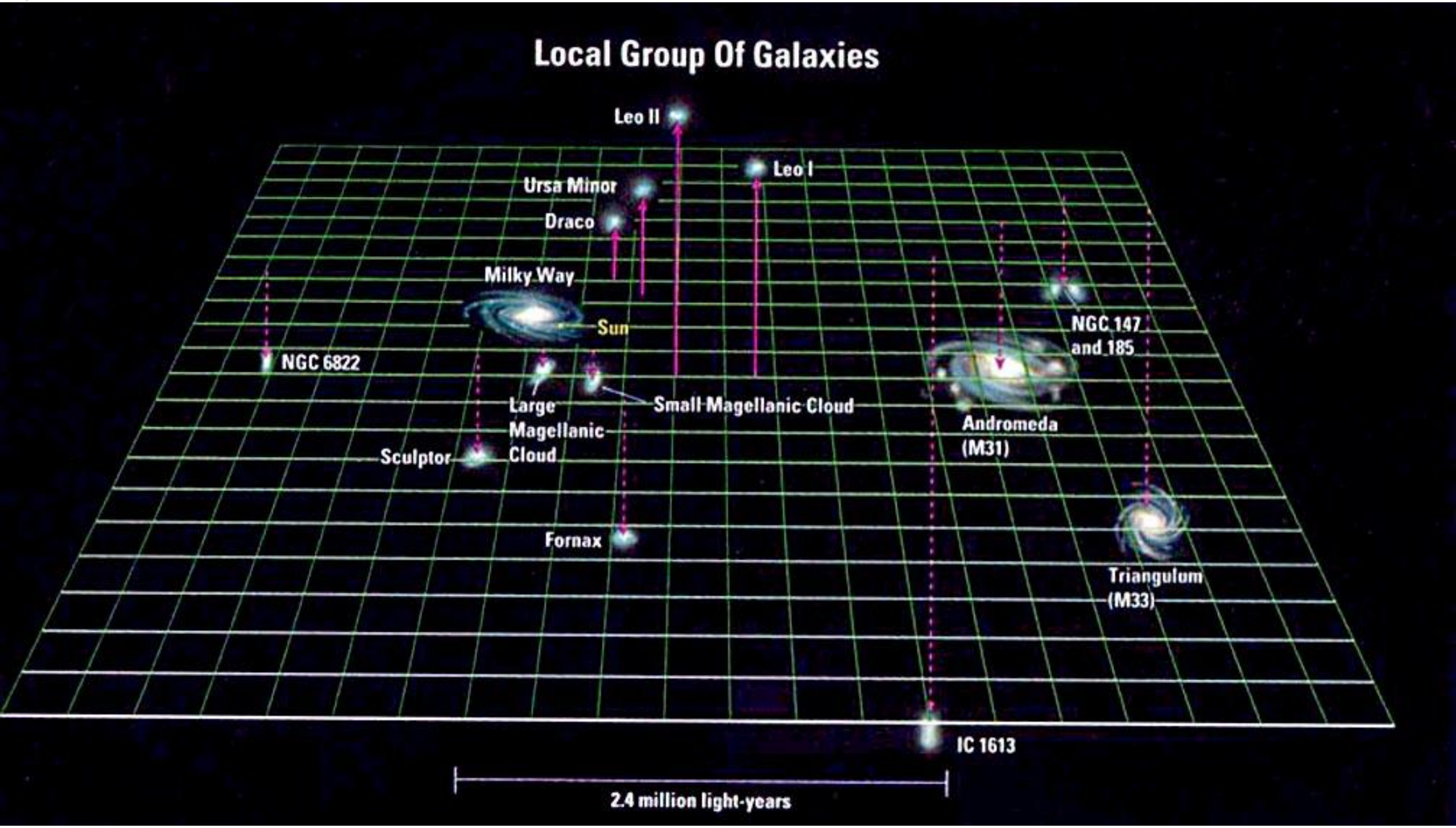
Local Group timing argument:

The most widely accepted interpretation of the negative velocity of M31 was first introduced by Kahn and Woltjer (1959). They assumed that the Milky Way and M31 system has negative energy, that means it is held together by gravitational forces.

Today, the argument of Kahn and Woltjer is considered as a proof for either the existence of large Dark Matter(DM) halos surrounding M31 and the Milky Way or (at least) a large common DM super halo pervading the Local Group.

They deduced, with a simple order of magnitude argument, that the effective mass was larger than $1.8 \times 10^{12}\ M_{\odot}$ about six times larger than the reduced mass of M31 and the Milky Way

T-11, Mass of the Local Group Galaxy



T-11, Mass of the Local Group Galaxy

Introduction to the problem

The basic idea is that galaxies currently in a binary system were at approximately the same point in space shortly after the big bang. Besides, mass of the local group is dominated by mass of Milky Way (MW) and M31.

We can measure the radial velocity of M31 with respect to MW via Doppler shifts of the spectral lines being found to be -118 km/s.

The negative sign means that M31 is moving towards MW. This may be surprising, given that most galaxies are moving apart with the general Hubble flow.

T-11, Mass of the Local Group Galaxy

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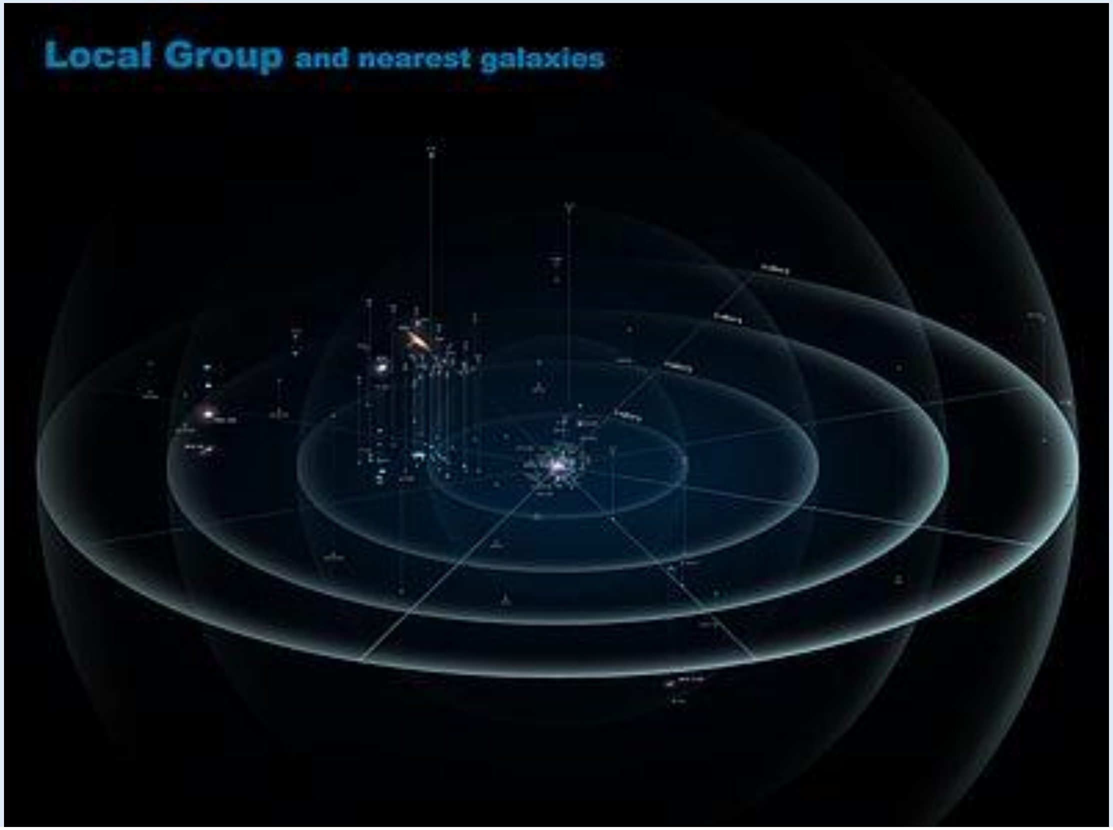
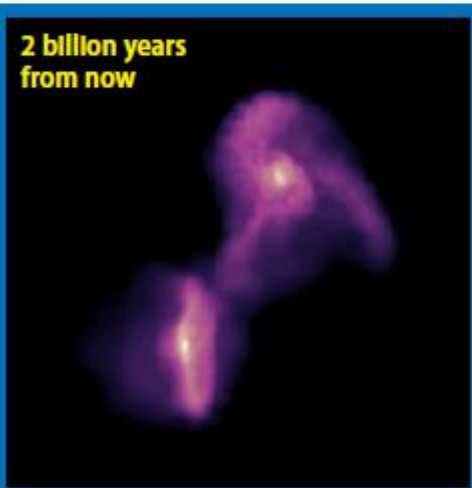
T-11, Mass of the Local Group Galaxy

Objectives:

To estimate the mass of the Local Group Galaxy from the dynamics of Milky Way - Andromeda System(M31)

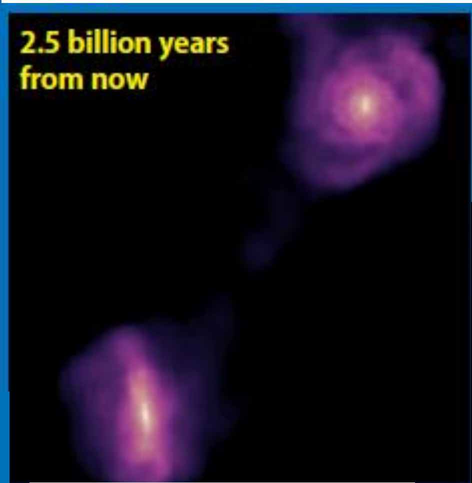


In courtesy of T.J Cox (Harvard Smithsonian)

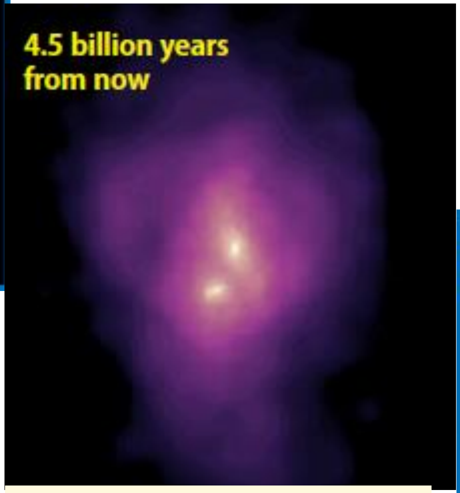


NGC 2207 (LEFT) merging with smaller IC 2163.

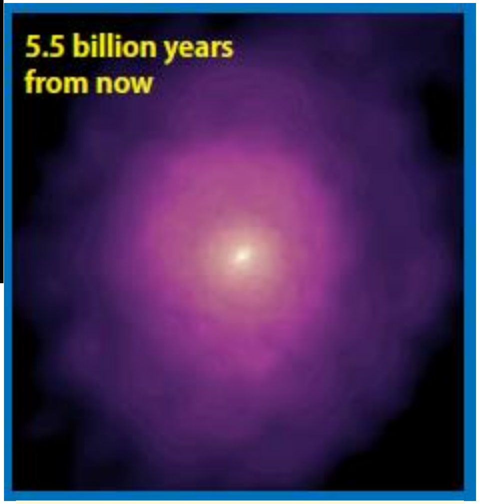
NASA/ESA/HUBBLE HERITAGE TEAMS (STSCI)



IN 2.5 BILLION YEARS, the galaxies are still moving apart. A ghostly bridge of gas and stars connects the galaxies. Stars in the bridge, perhaps some with planets, could end up literally lost of intergalactic space bridge dissipates.



IN 4.5 BILLION YEARS, the galax-ies loop around again and come back together to finally merge. Their dense cores, each harborin a supermassive black hole, gradu ally combine. The merging galax-ies experience a brief pulse of sta formation as the two black holes merge.

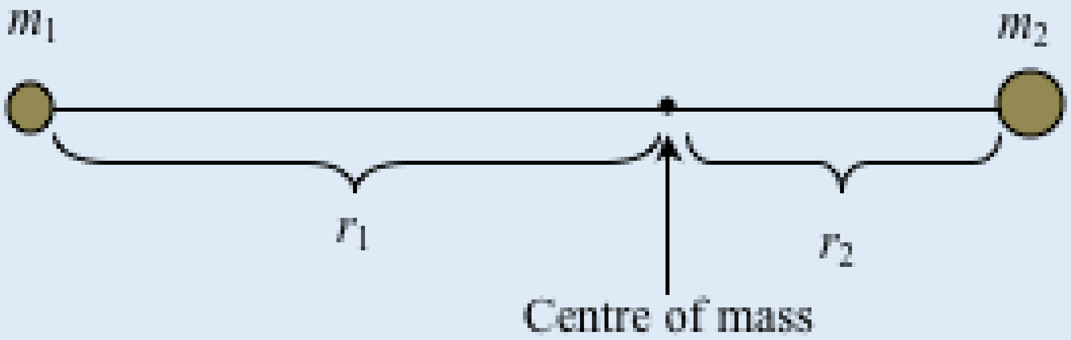


IN 5.5 BILLION YEARS, Milkom-eda is born. Tidal swirls,tails, and eddies left over from the violent merger slowly relax and dissipate. Individual stars spread out, forming a more smooth, internally homogenous elliptical galaxy similalr to the barrEd elliptical galaxy IN CEN-TAURUS NGC 2207 at right.

T-11, Mass of the Local Group Galaxy

In order to get the mass of LG:

- First we consider a non-rotating isolated system of two gravitating point masses m_1 , m_2 (as observed by an inertial observer at the centre of mass).



We express the total mechanical energy (E) of this system in a mathematical form connecting m_1, m_2, r_1, r_2, v_{r_1}, v_{r_1} and the universal gravitational constant G where v_{r_1}, v_{r_1} are radial velocities of m_1, m_2 respectively.

The total energy (E) is a negative quantity, therefore

$$\frac{1}{2}m_1 v_{r_1}^2 + \frac{1}{2}m_1 v_{r_2}^2 - \frac{Gm_1m_2}{r_1+r_2} = E \quad \text{-----------(1)}$$

$$m_1r_1 = m_2r_2, r = r_1 + r_2 \text{ and we have}$$

$$r_1 = \frac{m_2}{m_1+m_2}r, \; r_2 = \frac{m_1}{m_1+m_2}r. \quad \text{-----------(2)}$$

Substituting (2) into (1), then we get:

T-11, Mass of the Local Group Galaxy

$$\frac{1}{2}v_r^2 - \frac{G(m_1+m_2)}{r} = \frac{E}{\mu} \quad \text{where} \quad \mu \equiv \frac{m_1 m_2}{m_1+m_2}, \quad M \equiv m_1+m_2$$

$$\frac{1}{2}v_r^2 - \frac{2GM}{r} = \frac{2E}{\mu} \qquad \text{-----------(3)}$$

$$\frac{1}{2}v_r^2 = \frac{2GM}{r} + \frac{2E}{\mu}$$

$$= (2GM)\left\{\frac{1}{r} + \frac{1}{\left(\frac{GM\mu}{E}\right)}\right\}$$

$$= (2GM)\left\{\frac{1}{r} - \frac{1}{r_0}\right\}, \; r_0 \equiv -\frac{GM\mu}{E} \qquad \text{-----------(4)}$$

T-11, Mass of the Local Group Galaxy

$$r = \frac{r_0}{2}(1-\cos\theta), \qquad \text{-----------(5)}$$

$$v_{r'} = \left(\frac{r_0}{2}\sin\theta\right)\omega$$

$$t = \left(\frac{r_0^3}{8GM}\right)^{1/2}(\theta - \sin\theta), \qquad \text{-----------(6)}$$

$$1 = \left(\frac{r_0^3}{8GM}\right)^{1/2}(1-\cos\theta)\ \omega$$

$$\therefore \frac{v_r t}{r} = \frac{\left(\frac{r_0}{2}\sin\theta\right)\left(\frac{r_0^3}{8GM}\right)^{1/2}(\theta - \sin\theta)}{\left(\frac{r_0^3}{8GM}\right)^{1/2}(1-\cos\theta)\frac{r_0}{2}(1-\cos\theta)} = \frac{(\sin\theta)(\theta-\sin\theta)}{(1-\cos\theta)^2}.$$

T-11, Mass of the Local Group Galaxy

$$v_r = -118 \text{ kms}^{-1} = -118\times10^3 \text{ ms}^{-1}$$

$$r = 710 \text{ kpc} = 2.1908\times10^{22} \text{ m}$$

$$t = 13700\times10^6 \text{ years} = 4.3233\times10^{17} \text{ s}$$

$$\therefore \frac{v_r t}{r} = -\frac{118\times10^3\times4.3233\times10^{17}}{2.1908\times10^{22}} = -2.329 = \frac{(\sin\theta)(\theta-\sin\theta)}{(1-\cos\theta)^2}$$

The negative value of the left-hand side of this equation implies that θ is greater than π.

T-11, Mass of the Local Group Galaxy

q	$(\sin q)(q - \sin q)/(1 - \cos q)^2$
$200° = 3.491$ radians	-0.348
$210° = 3.665$ radians	-0.598
$240° = 4.189$ radians	-1.946
$244° = 4.259$ radians	-2.241
$245° = 4.276$ radians	-2.321
$246° = 4.294$ radians	-2.404
$250° = 4.363$ radians	-2.768

This gives the value of $\theta = 245° = 4.276$ radian

T-11, Mass of the Local Group Galaxy

From equation (5) we have $r_{max} = r_0$, and hence

$$r_{max} = \frac{2r(\theta)}{1-\cos\theta} = \frac{2\times 710\text{ kpc}}{1-\cos 245^\circ} = 998\text{ kpc}$$

(This is the maximum separation between Our Milky-Way Galaxy and Andromeda Galaxy, afterwards which they started to move towards each other.)

The value of M may be calculated from the equation (5)

$$M = \left(\frac{r_{max}^3}{8G}\right)\left\{\frac{\theta - \sin\theta}{t(\theta)}\right\}^2.$$

$$r_{max} = 998\text{ kpc} = 3.0795\times 10^{22}\text{ m}$$

$$\theta = 245^\circ = 4.276\text{ radians}$$

$$t = t(\theta) = 13700\times 10^6\text{ years} = 4.3233\times 10^{17}\text{ s}$$

T-11, Mass of the Local Group Galaxy

$$G = 6.6726\times10^{-11}\ \mathrm{Nm^2kg^{-2}}$$

$$\therefore M = 7.861\times10^{42}\ \mathrm{kg} = \frac{7.861\times10^{42}}{1.9891\times10^{30}} M_{\odot} = 3.952\times10^{12} M_{\odot}$$

The estimated number of stars, most of which are of less than a solar mass, for Andromeda and Our Galaxy are 4×10^{11} and 2×10^{11} respectively. This implies that most of mass of the system is DARK.