Long Question: T-13

“Exomoon”

Motivation

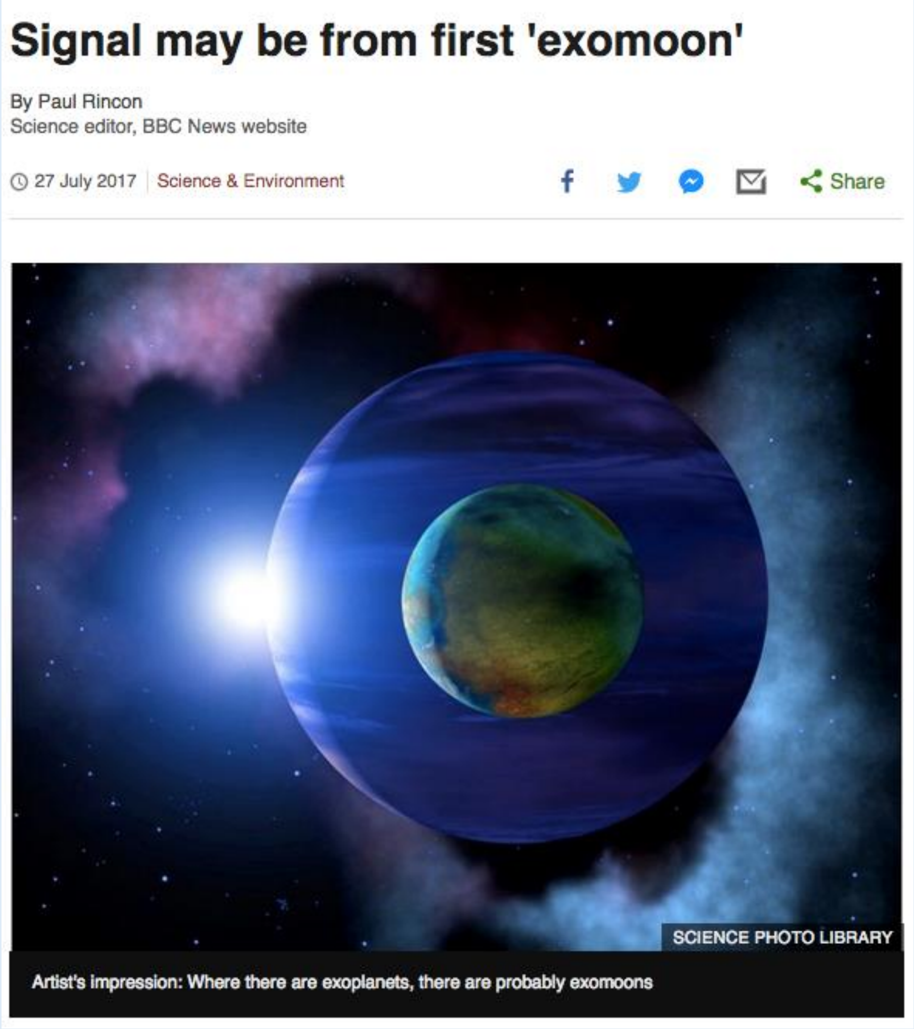
- Exomoon
  - Exomoon is a natural satellite of exoplanet

T-13, Exomoon

Motivation

- Kepler-1625 b I, an exomoon candidate, orbits a Neptune-size gas giant roughly 4,000 light years from Earth.
  (Teachey et al. 2017)

- Several methods have been developed in order to detect exomoons, including:
  - TTV → Planet position around planet-moon barycentre
  - TDV → Planet velocity around planet-moon barycentre



Signal may be from first 'exomoon'

By Paul Rincon
Science editor, BBC News website

27 July 2017 | Science & Environment

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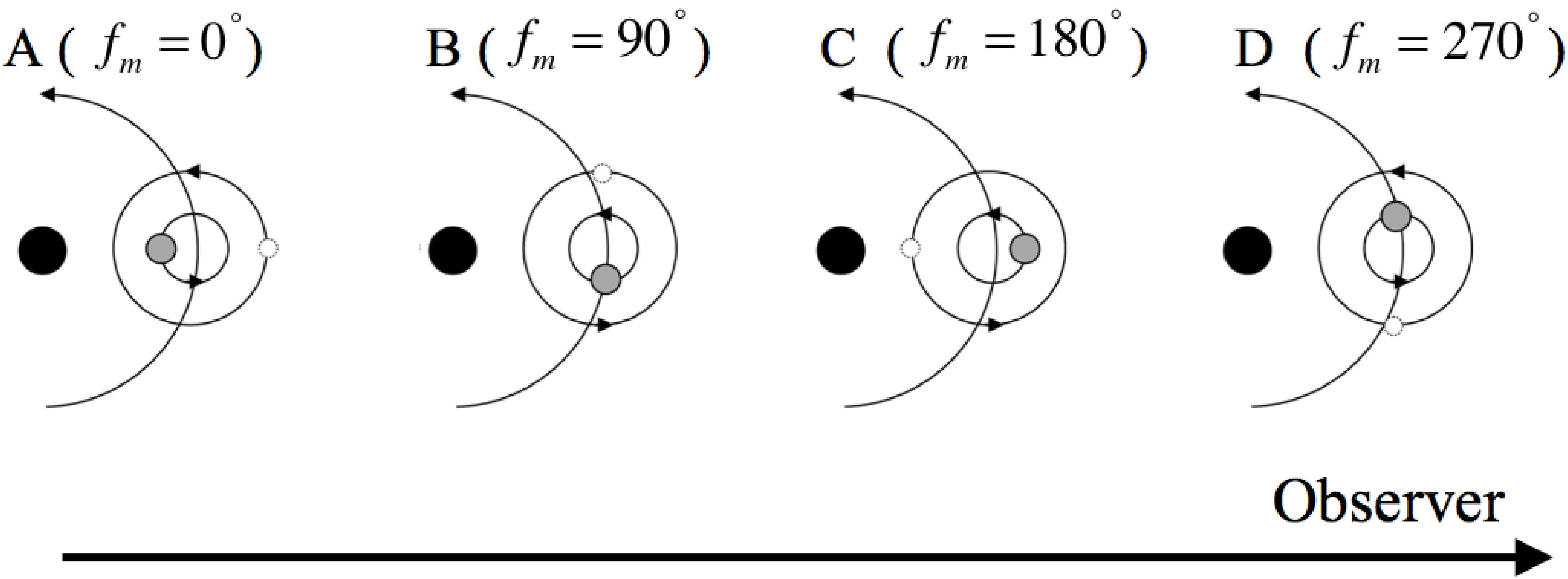
Artist's impression: Where there are exoplanets, there are probably exomoons

[Credit: BBC]

Motivation

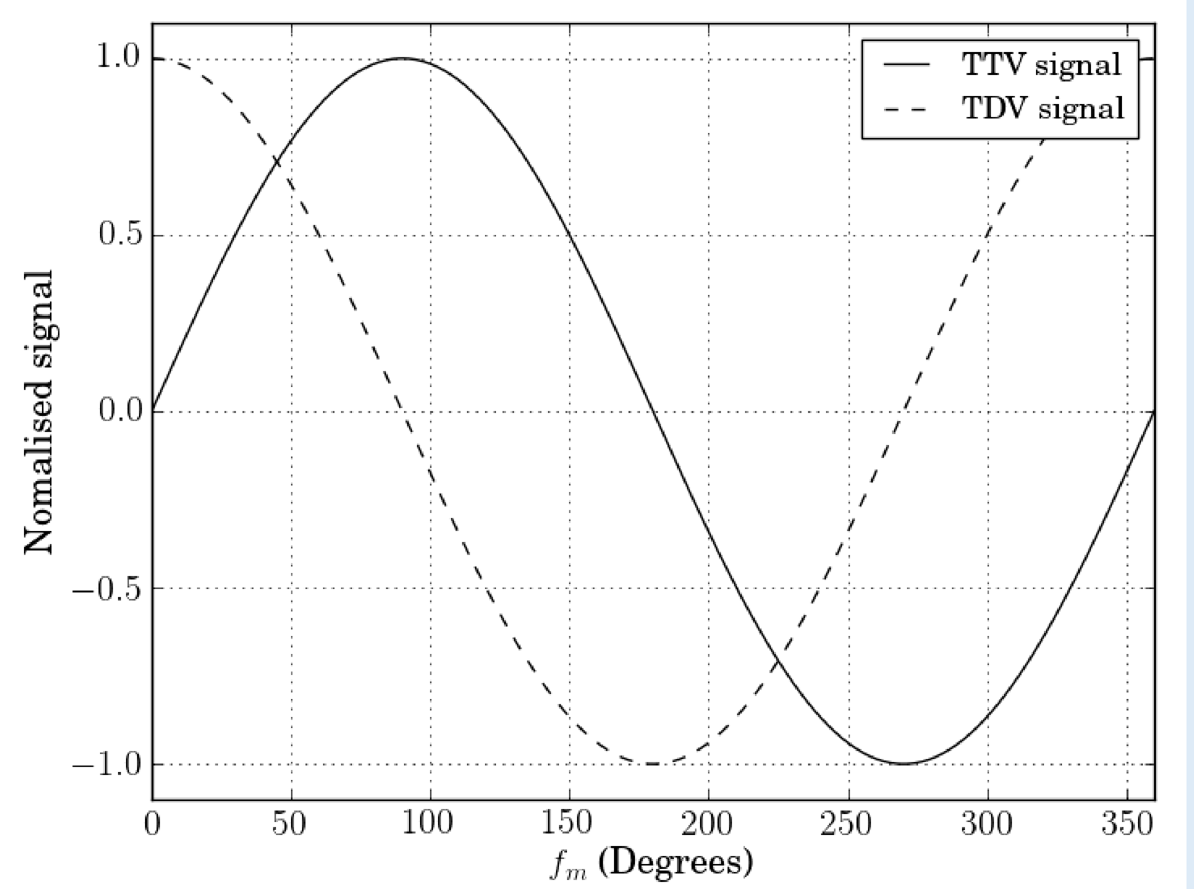
- f_m is moon phase
  - $f_m = 0^\circ$ when the moon is in opposition to the star



Motivation

$$\sigma_{TTV} = \left[\frac{a_m M_m P_p}{2\pi a_p M_p}\right]\sin(f_m)$$

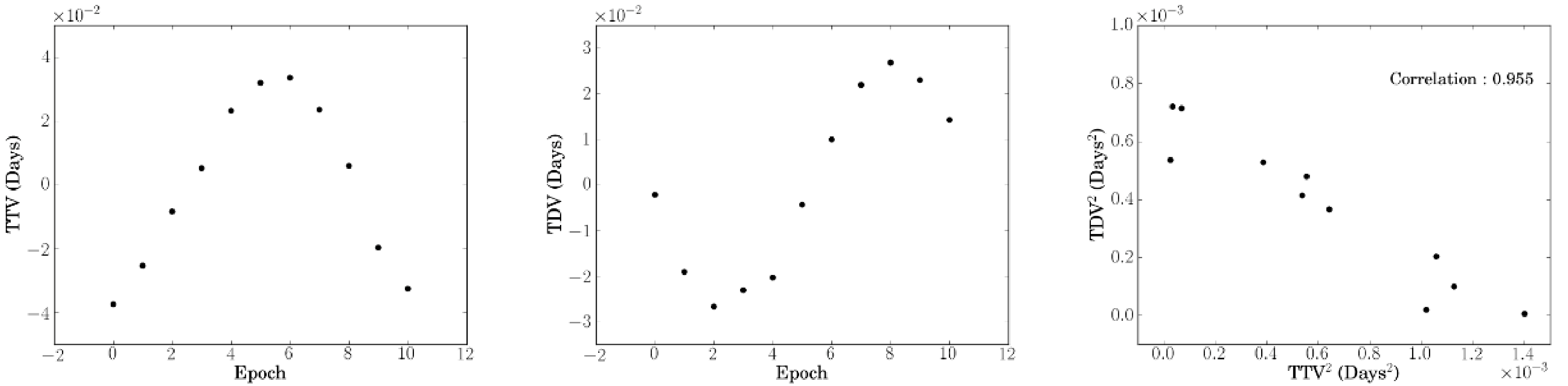
$$\sigma_{TDV} = \tau\left[\frac{P_p M_m a_m}{P_m M_p a_p}\right]\cos(f_m)$$



Motivation

- The relation between TTV signal (σ_{TTV}^2)and TDV signal (σ_{TDV}^2) can be written as a linear function (Awiphan et al. 2013)

$$\sigma_{TDV}^2 = -\left(\frac{2\pi\tau}{P_m}\right)^2 \sigma_{TTV}^2 + \tau^2 \left(\frac{a_m M_m P_p}{a_p M_p P_m}\right)^2$$

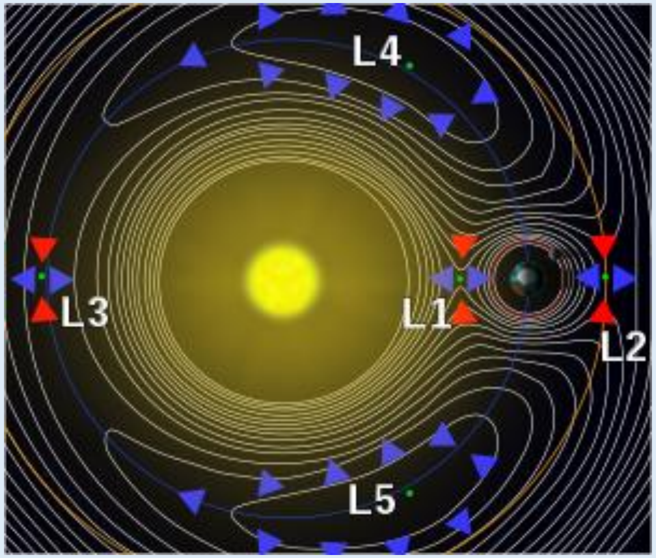


T-13, Exomoon

Motivation

- **Hill sphere**
  - A region around a planet within which the planet's gravity dominates (Stable orbit).
  - For massive host star, the radius of the Hill sphere of the system is approximately equal to the distance between planet and Lagrange point L_1 or L_2

$$R_h = a_p \sqrt[3]{\frac{M_p}{3M_*}}$$

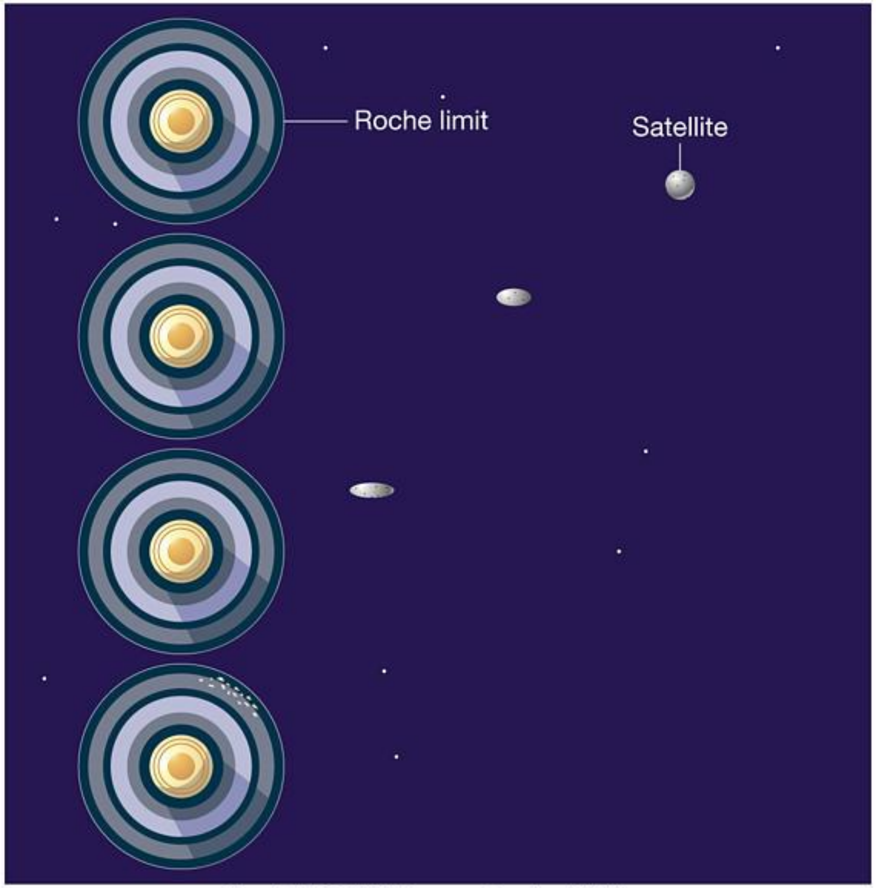


A contour plot of the effective potential of a two-body system and Lagrange points [Credit: Wikipedia]

Motivation

- **Roche limit**
  - The minimum orbital radius which a satellite can orbit without being torn apart by tidal force.

$$R_r = 2.44 R_p \sqrt[3]{\frac{r_p}{r_m}}$$



Roche limit [Credit: Pearson Prentice Hall, Inc]

Objectives

The ultimate goal is **to find that the exomoon has a stable orbit or not.**

1. Solve equations of TTV and TDV signals
2. Find exomoon parameters
3. Find Hill radius and Roche limit radius of the system
4. Does the exomoon have a stable orbit?

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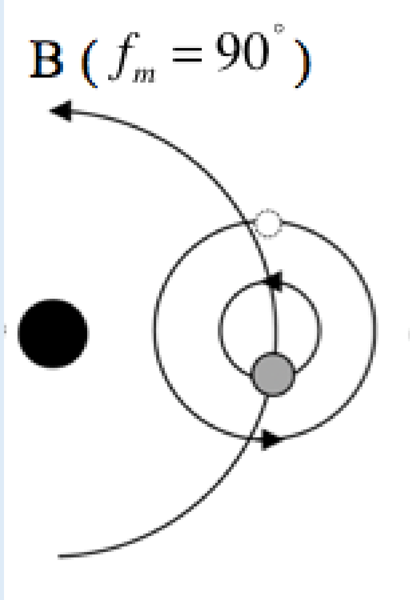
Task 1 (7 marks)

- Show that the TTV signal can be written as,

$$\sigma_{TTV} = \left[\frac{a_m M_m P_p}{2\pi a_p M_p} \right] \sin(f_m)$$

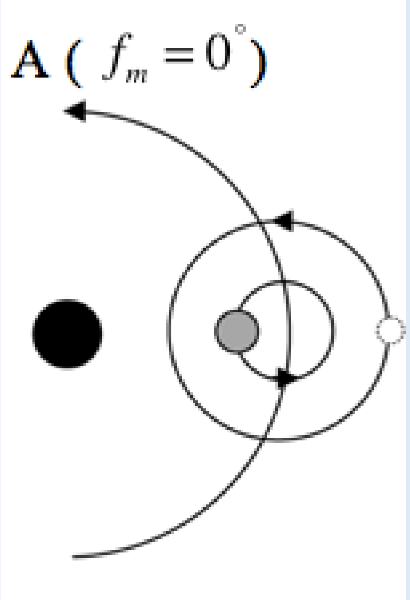


- Projection distance of the planet to the planet-moon barycentre

$$a_m = \frac{M_p}{M_m} a_{pb} \qquad a_{proj} = a_{pb} \sin(f_m) \qquad s_{TTV} = \left(\frac{a_{proj}}{a_p} \right) \Big/ \left(\frac{2p}{P_p} \right)$$

Task 2 (9 marks)

- Show that the TDV signal can be written as,



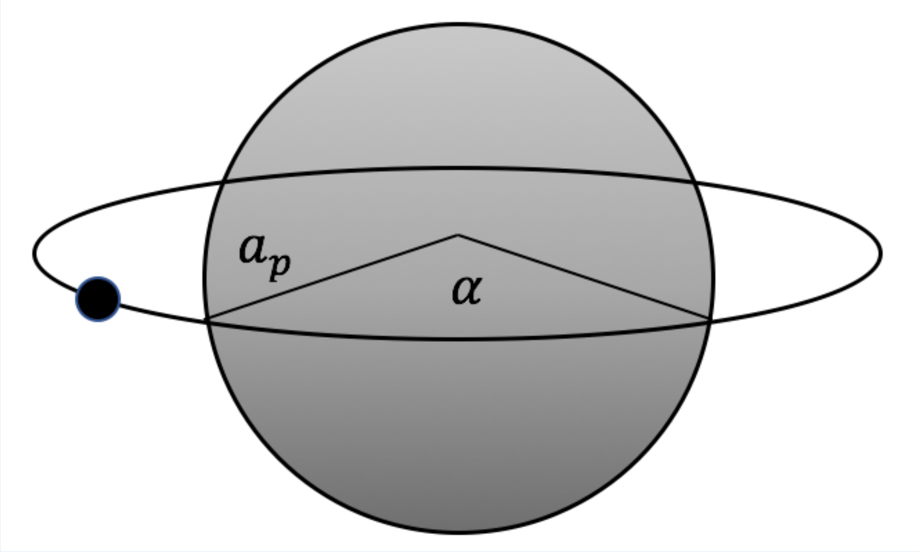
$$\sigma_{TDV} = \tau \left[\frac{P_p M_m a_m}{P_m M_p a_p} \right] \cos(f_m)$$

- Transverse velocity of planet around the planet-moon barycentre

$$v_{pb} = \frac{2\pi}{P_m} a_{pb} \qquad v_{trans} = -\frac{2\pi M_m}{P_m M_p} a_m \cos(f_m) \qquad \sigma_{TDV} = \tau \left[\frac{v_{trans}}{v_p} \right]$$

Task 3 (5 marks)

- Find mean transit duration
- Students need to determine
  - Distance of the planet-moon barycentre to the star
  - Mean transit duration



$$P^2 = \frac{4\pi^2}{G(M_* + M_p)} a_p^3$$

$$\tau = P \frac{\alpha}{2\pi}$$

Task 4 (6 marks)

- Find moon period
  - Linear function of TTV (σ^2_{TTV}) and TDV signal (σ^2_{TDV})

$$\sigma^2_{TDV} = -0.7432\sigma^2_{TTV} + 1.933\times10^{-8}\ \text{days}^2$$

$$\sigma^2_{TDV} = -\left(\frac{2\pi\tau}{P_m}\right)^2 \sigma^2_{TTV} + \tau^2\left(\frac{a_m M_m P_p}{a_p M_p P_m}\right)^2$$

$$-\left(\frac{2\pi\tau}{P_m}\right)^2 = -0.7432$$

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Task 5 (3 marks)

- **Find distance of the moon to the planet-moon barycentre**
- Kepler's third law

$$P_m^2 = \frac{4\pi^2}{GM_p} a_m^3$$

Task 6 (4 marks)

- **Find moon mass**
- Linear function of TTV (σ_{TTV}^2) and TDV signal (σ_{TDV}^2) *[Task 4]*
- Distance of the planet-moon barycentre to the star and transit durataion *[Task 3]*
- Distance of the moon to the planet-moon barycentre *[Task 5]*

$$\sigma_{TDV}^2 = -0.7432\sigma_{TTV}^2 + 1.933\times10^{-8}\ \text{days}^2 \qquad \tau^2 \left(\frac{a_m M_m P_p}{a_p M_p P_m}\right)^2 = 1.933\times10^{-8}\ \text{days}^2$$

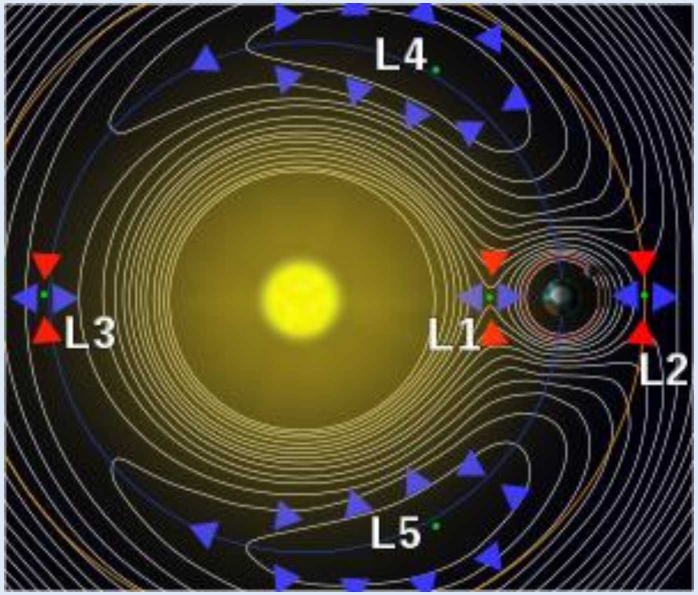
Task 7 (10 marks)

- **Find radius of Hill sphere**
- **Students need to determine**
  - Hill sphere function from Lagrange point

$$R_h = a_p \sqrt[3]{\frac{M_p}{xM_*}} \qquad a_m = a_p \sqrt[3]{\frac{M_p}{3M_*}} \qquad x = 3$$

  - Hill sphere radius

$$R_h = 52.2 R_\oplus$$



A contour plot of the effective potential of a two-body system and Lagrange points [Credit: Wikipedia]

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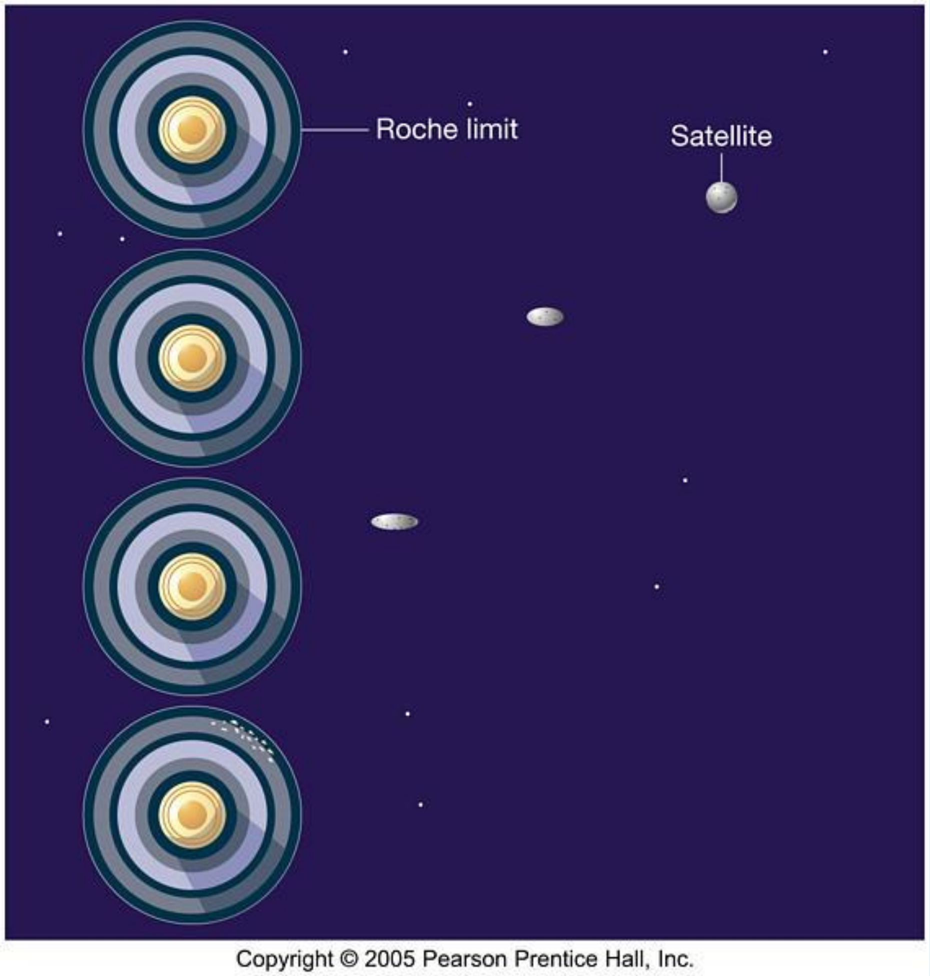
Task 8 (3 marks)

- **Find radius of Roche limit**
- Roche limit radius

$$R_r = 2.44 R_p \sqrt[3]{\frac{r_p}{r_m}}$$

$$R_r = 12.0 R_\oplus$$



Roche limit [Credit: Pearson Prentice Hall, Inc]

T-13, Exomoon

Task 9 (3 marks)

- Does the exomoon have a stable orbit?
- **Students need to use**
  - Distance between moon and planet-moon barycentre *[Task 5]*
  - Hill radius *[Task 7]*
  - Roche limit *[Task 8]*

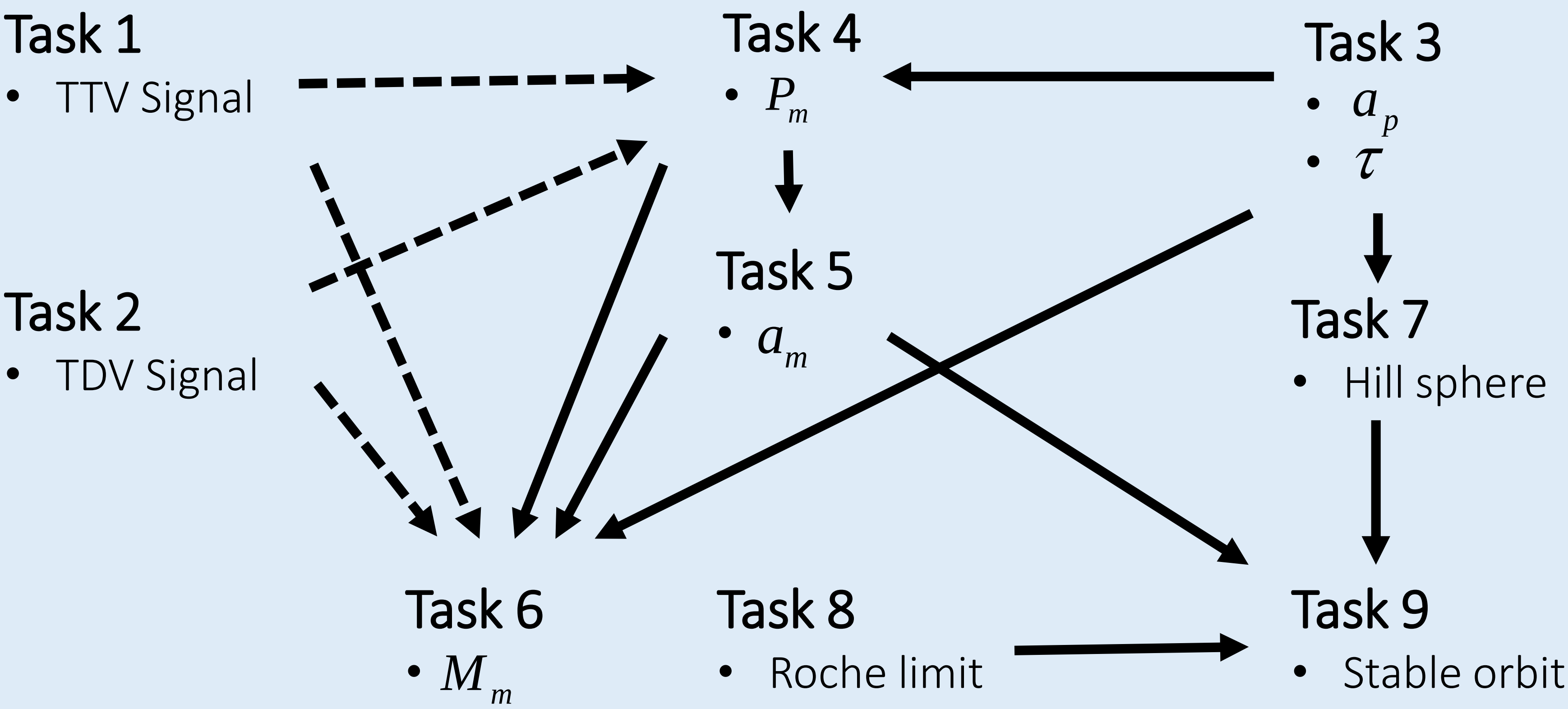
$$R_r < a_m < R_H$$

Stable orbit

Knowledge

- **Basic Astrophysics/ Celestial Mechanics:**
  - Newton's laws of gravitation
  - Kepler's law for circular and non-circular orbits
  - Roche limit
  - Barycentre
  - 2-body problem
  - Lagrange points
- **Stellar system/ Exoplanet:**
  - Techniques used to detect exoplanet

Modularity


Task 1
• TTV Signal
Task 4
• P_m
Task 3
• a_p
• τ
Task 2
• TDV Signal
Task 5
• a_m
Task 7
• Hill sphere
Task 6
• M_m
Task 8
• Roche limit
Task 9
• Stable orbit