1. Photometry and spectroscopy of Nova Del 2013

60 p

a) From the light curve plot the Modified Julian Dates can be read with an error of about 0.5. The peak of the maximum brightness is very narrow and obviously placed at $\mathrm{MJD}_0 = 56\,520.5$ with a value of $m_0 = 4.5^\mathrm{m}$, therefore:

$$\mathrm{MJD}_0 = \boxed{56\,520.5 \pm 0.5} \qquad (2\,\mathrm{p})$$

$$m_0 = \boxed{4.5^\mathrm{m} \pm 0.05^\mathrm{m}} \qquad (1\,\mathrm{p})$$

b) The brightness values of 2^m and 3^m decline are $6.5^\mathrm{m} \pm 0.05^\mathrm{m}$ and $7.5^\mathrm{m} \pm 0.05^\mathrm{m}$, respectively. (2 p)

The corresponding Modified Julian Dates are MJD_2 and MJD_3. Because of the poorly defined slopes on the light curve around these dates, their acceptable error is larger than in other parts of the light curve, let say it is about 1^d, so:

$$\mathrm{MJD}_2 = \boxed{56\,531.5 \pm 1}, \quad \mathrm{MJD}_3 = \boxed{56\,543.5 \pm 1} \qquad (2\,\mathrm{p})$$

$$t_2 = \boxed{11^\mathrm{d} \pm 1^\mathrm{d}}, \quad t_3 = \boxed{23^\mathrm{d} \pm 1^\mathrm{d}} \qquad (2\,\mathrm{p})$$

c) The text of this part does not ask for calculating the individual errors of the formulae, but it is worth estimating them here, just for the sake of completeness. (Students won't do it.)

(a) The form of the function is

$$(1.1)\quad M = a + b \arctan \frac{c - \log t_2}{d}, \quad a = -7.92, b = -0.81, c = 1.32, d = 0.23,$$

so its derivative:

$$(1.2)\quad M' = -\frac{b}{d \log(10) \left[1 + \left(\frac{c - \log t_2}{d}\right)^2\right]} \frac{1}{t_2} \to \Delta M = -\frac{b}{d \log(10) \left[1 + \left(\frac{c - \log t_2}{d}\right)^2\right]} \frac{\Delta t_2}{t_2}$$

The value and its error calculated from the formulae above (error is not necessary):

$$M_0^{(a)} = \boxed{-8.63^\mathrm{m} \pm 0.06^\mathrm{m}} \qquad (1\,\mathrm{p})$$

(b) The form of the function is

$$(1.3)\quad M = a + b \log t_2, \quad a = -11.32, b = 2.55$$

so its derivative:

$$(1.4)\quad M' = \frac{b}{t_2 \log(10)} \to \Delta M = \frac{b}{\log(10)} \frac{\Delta t_2}{t_2}$$

The value and its error calculated from the formulae above (error is not necessary):

$$M_0^{(b)} = \boxed{-8.66^\mathrm{m} \pm 0.10^\mathrm{m}} \qquad (1\,\mathrm{p})$$

(c) The form of the function is

$$(1.5)\quad M = a + b \log t_3, \quad a = -11.99, b = 2.54$$

so its derivative:

$$(1.6)\quad M' = \frac{b}{t_3 \log(10)} \to \Delta M = \frac{b}{\log(10)} \frac{\Delta t_3}{t_3}$$

The value and its error calculated from the formulae above (error is not necessary):

$$M_0^{(c)} = \boxed{-8.53^\mathrm{m} \pm 0.05^\mathrm{m}} \qquad (1\,\mathrm{p})$$

The standard deviation of a dataset can be calculated as

$$(1.7)\quad \sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}},$$

where n is the number of data points, x_i is the i^{th} individual data value and $\bar{x}$ is their mean, $\bar{x} = (x_1 + x_2 + \ldots + x_n)/n$.

The mean and the standard deviation of the three absolute maximum brightness values:

$$M_0 = \boxed{-8.61^{\text{m}} \pm 0.07^{\text{m}}}$$ (2 p)

d) The color excess $E(B-V)$ is the difference between the observed color index of the star and the intrinsic color index predicted from its spectral type:

$$(1.8)\quad E(B-V) = (B-V) - (B-V)_0 = A_B - A_V$$

The total extinction is quantified by A_V (at 5550 Å). The ratio of total-to-selective extinction:

$$(1.9)\quad R = \frac{A_V}{E(B-V)} \rightarrow A_V = RE(B-V), \text{ where } R = 3.1$$ (2 p)

With the given value and error of $E(B-V)$:

$$A_V = 3.1 \times E(B-V) = 3.1 \times (0.184^{\text{m}} \pm 0.035^{\text{m}}) = \boxed{0.57^{\text{m}} \pm 0.11^{\text{m}}}$$ (2 p)

e) According to the formula for the distance modulus:

$$(1.10)\quad m_V - M_V = -5 + 5\log d + A_V \rightarrow$$ (1 p)

$$(1.11)\quad \log d = \frac{m_V - M_V + 5 - A_V}{5} \rightarrow$$

$$(1.12)\quad d = 10^{(m_V - M_V + 5 - A_V)/5},$$ (2 p)

where the distance d is in parsecs.

Since $\Delta a^x / \Delta x = a^x \ln a$, therefore the error of the distance d:

$$\Delta d = 10^{(m_V - M_V + 5 - A_V)/5} \times \ln 10 \times \Delta((m_V - M_V + 5 - A_V)/5)$$ (2 p)

The error of $(m_V - M_V + 5 - A_V)/5$ can be estimated with the sum of the errors of m_V, M_V and A_V, so:

$$\Delta((m_V - M_V + 5 - A_V)/5) \approx 0.05$$ (2 p)

With the data:

$$d = 3220\,\text{pc and } \Delta d = 338\,\text{pc},$$ (2 p)

so the distance to the nova:

$$d \approx \boxed{3.2 \pm 0.3\,\text{kpc}}$$ (2 p)

f) The well known Doppler formula between the wavelength displacement and radial velocity:

$$\text{(1.13)} \quad \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} = \frac{v_\mathrm{r}}{c} \rightarrow v_\mathrm{r} = \frac{\Delta\lambda}{\lambda_0}\,c, \qquad (1\,\mathrm{p})$$

where λ is the measured wavelength of the line feature, λ_0 is the rest wavelength of the line, c is the speed of light, and v_r is the radial velocity to be calculated.

The wavelength of the P Cygni absorption peak should be extracted from the figure with an error of about 1 Å. The main part of this error is coming from the "definition" of the peak position of the Gaussian-like profile. This will result in inaccuracy of about $\Delta v_\mathrm{r} = \pm 50\,\mathrm{km\,s^{-1}}$ in radial velocities. (1 p)

The wavelengths and radial velocities should be something like these:

MJD	WL	RV
56518.986	6527	-1636
56519.813	6531	-1454
56520.843	6534	-1317
56521.835	6537	-1179
56522.829	6542	-951
56523.827	6544	-860

6 points for the wavelength values and 6 points for the radial velocity values. (12 p)

Radial velocities within the range of $\pm 50\,\mathrm{km\,s^{-1}}$ of the RV values listed in the table should be given full marks, but velocities in the range of $\pm 100\,\mathrm{km\,s^{-1}}$ are still acceptable with half marks.

g) See the attached figure as an example for the acceptable solution. The plotted data are taken from the table above. For the sake of simplicity the absolute values of the radial velocities have been used for making the graph. (6 p)

h) It is obvious from the plot, that the radial velocities lie along a straight line.

To estimate the size of the expanding envelope we need to calculate the area below the $t - v_\mathrm{r}$ graph between the first and last date.

Hence the graph is a straight line, this is very simple: we have to determine the area of the hatched region which is a trapezoid.

If the two bases and the height of the trapezoid are a, b, and m, respectively, then the area of the trapezoid is:

$$\text{(1.14)} \quad T = \frac{a+b}{2}\,m \qquad (3\,\mathrm{p})$$

In our case $a = v_{\mathrm{r}_1}$, $b = v_{\mathrm{r}_6}$, and $m = t_6 - t_1$. (1 p)

We could use the fitted line (dashed) as the upper side of the trapezoid, but this would be a bit complicated – because of the difficulties of the fitting process –, and not necessary at all. Instead of this we use the line connecting the first and last radial velocity points as this runs very close to the fitted line.

The result: $R \approx \boxed{3.5\,\mathrm{AU}}$ (3 p)

i) The apparent angular diameter of the spherical envelope seen from the Earth:

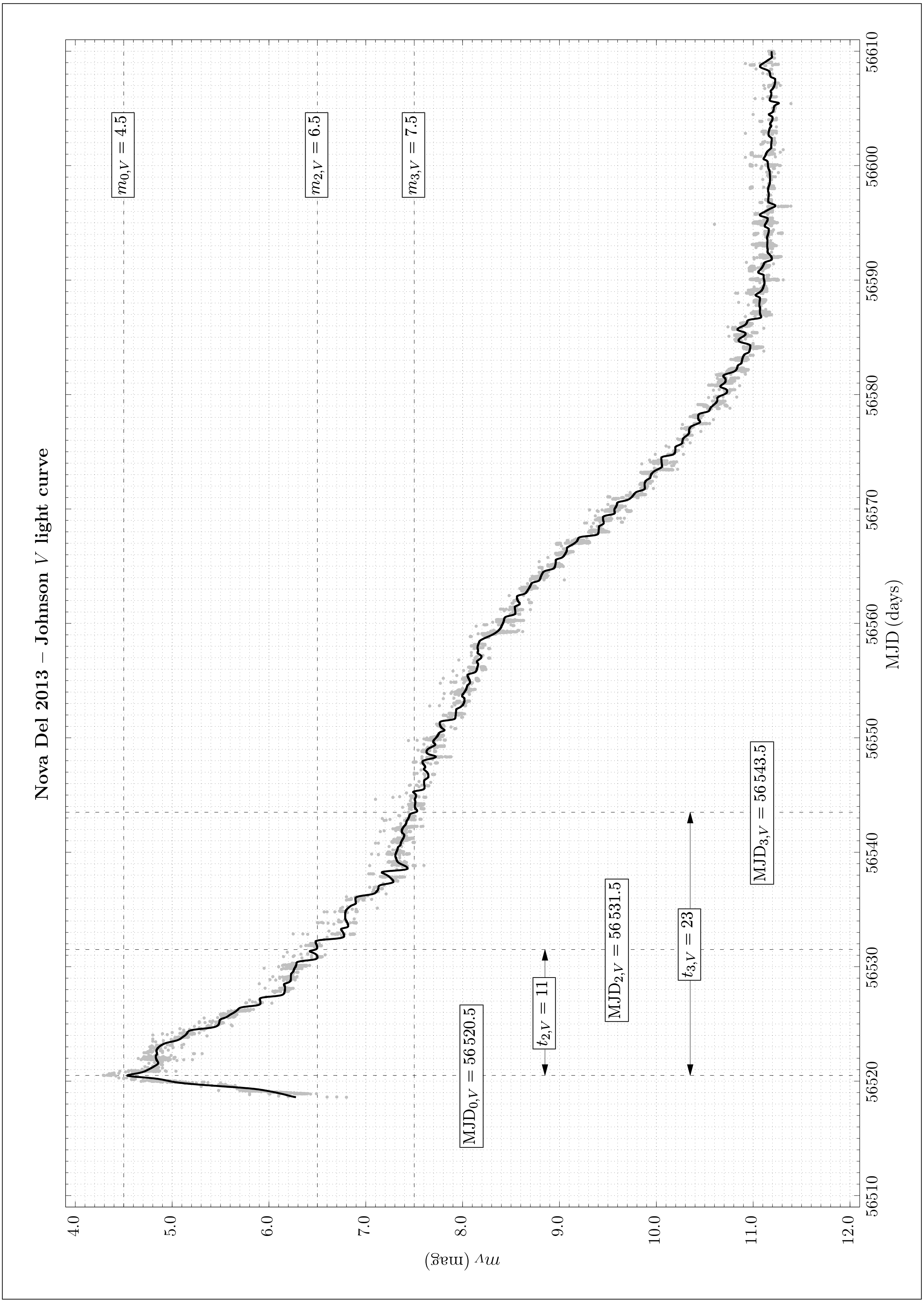
$$\text{(1.15)} \quad \vartheta = 2 \times \arctan\left(\frac{R}{d}\right) \approx 2\,\frac{R}{d} \qquad (2\,\mathrm{p})$$

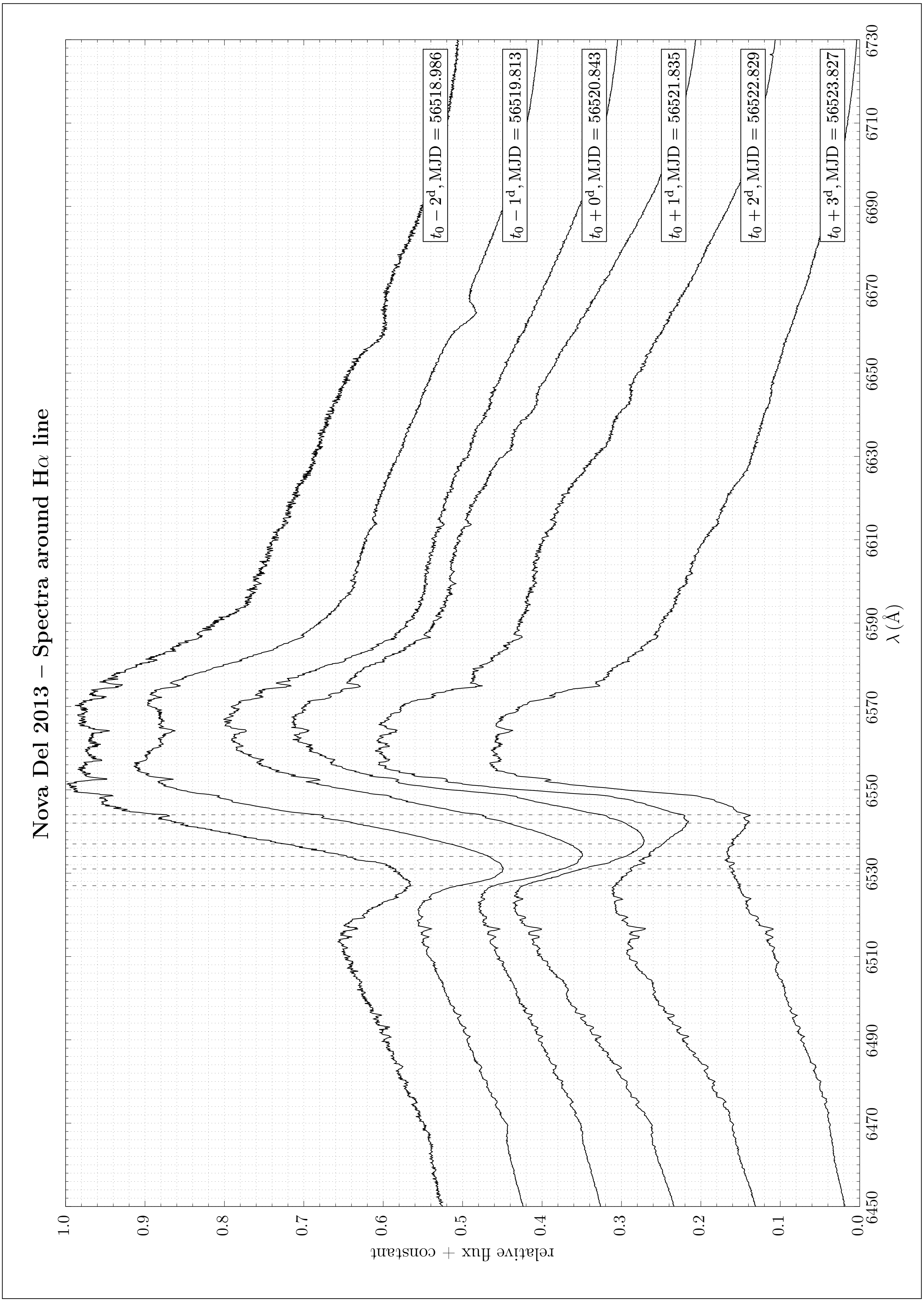
Using the values of $d \approx 3.2\,\text{kpc}$ and $R \approx 3.5\,\text{AU}$, 5 days after the discovery the angular diameter of the envelope is:

$$\vartheta = \boxed{0.0022''} = \boxed{2.2\,\text{mas}} \qquad (2\,\text{p})$$

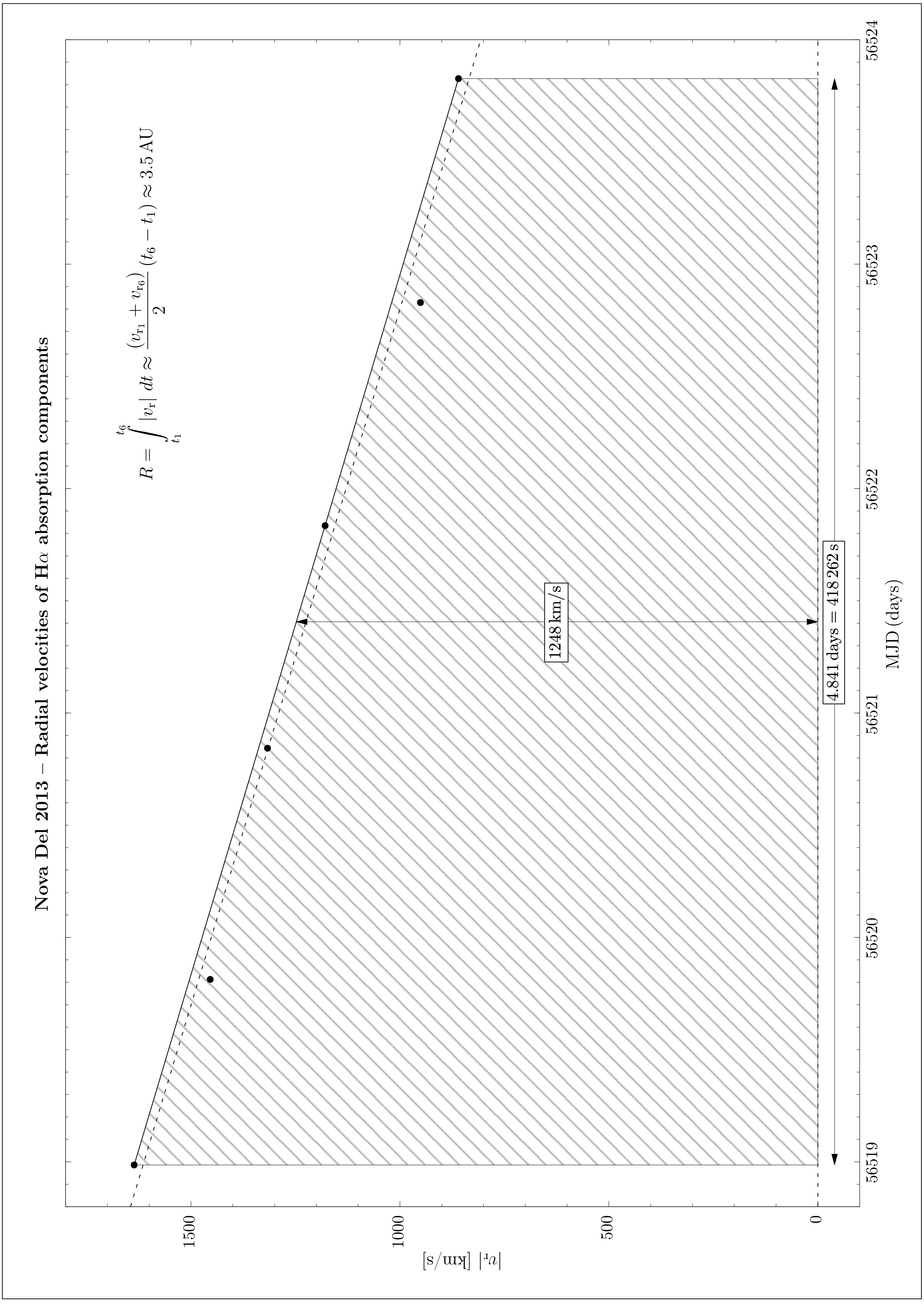
A less formal solution:

- By definition a parsec (1 pc) is the distance from the Sun to an astronomical object that has a parallax angle of one arcsecond, i.e. it represents the distance at which the radius of Earth's orbit (1 AU) subtends an angle of one arcsecond.
- Because of the very small angles the distance is a linear function of the parallax. This means that the radius $R \approx 3.5\,\text{AU}$ of the envelope subtends one arcsecond viewing from a distance of $d \approx 3.5\,\text{pc}$, and one milliarcsecond from a distance of $1000d \approx 3500\,\text{pc} = 3.5\,\text{kpc}$.
- Since this value is close to the distance of Nova Del 2013 determined earlier, we can conclude that the apparent angular diameter of the spherical shape envelope 5 days after the discovery was about 2 milliarcseconds.


Nova Del 2013 – Johnson V light curve
$m_{0,V} = 4.5$
$m_{2,V} = 6.5$
$m_{3,V} = 7.5$
$\mathrm{MJD}_{0,V} = 56\,520.5$
$t_{2,V} = 11$
$\mathrm{MJD}_{2,V} = 56\,531.5$
$t_{3,V} = 23$
$\mathrm{MJD}_{3,V} = 56\,543.5$
MJD (days)
m_V (mag)
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0
56510
56520
56530
56540
56550
56560
56570
56580
56590
56600
56610


Nova Del 2013 – Spectra around Hα line
t0 − 2d, MJD = 56518.986
t0 − 1d, MJD = 56519.813
t0 + 0d, MJD = 56520.843
t0 + 1d, MJD = 56521.835
t0 + 2d, MJD = 56522.829
t0 + 3d, MJD = 56523.827
relative flux + constant
λ (Å)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
6450
6470
6490
6510
6530
6550
6570
6590
6610
6630
6650
6670
6690
6710
6730

Data analysis



2. Triply eclipsing hierarchical triple stellar system

90 p

i) a) See the following table. (10 p)

event no.	contacts	components	BJD	φ_1	φ_2
1	I	A, B	2455476.1096	0.1226	0.9747
	II	A, C	2455476.4245	0.4703	0.9816
	III	A, B	2455477.9677	0.1743	0.0155
	IV	A, B	2455478.4722	0.7313	0.0266
2	I	A, B	2455521.5217	0.2643	0.9734
3	III	A, C	2455568.9434	0.6248	0.0163
4	I	A, C	2455612.4733	0.6882	0.9736
	III	A, C	2455614.3571	0.7682	0.0150
5	III	A, B	2455659.9241	0.0808	0.0171
	IV	A, C	2455660.2422	0.4320	0.0241

b) See the following table. (5 p)

event no.	closer component
1	A
2	A
3	A
4	A
5	A

All these events occur close to $\varphi_2 = 0$ (or $\varphi_2 = 1$) which means by definition that star A eclipses stars B and C. Therefore, star A is closer to the observer.

For full mark there is no need for explanation. 1 point for each event with correct answer.

c) In the moments of the 1st and last (4th) contacts:

(2.1) $$R_\mathrm{A} + R_\mathrm{B,C} = d_\mathrm{A\text{–}B,C},$$ (1 p)

while in the case of the 2nd and 3rd (inner) contacts:

(2.2) $$R_\mathrm{A} - R_\mathrm{B,C} = d_\mathrm{A\text{–}B,C},$$ (1 p)

where $d_\mathrm{A\text{–}B,C}$ stands for the sky-projected distance of the disks of the occulting star A and occulted component B or C.

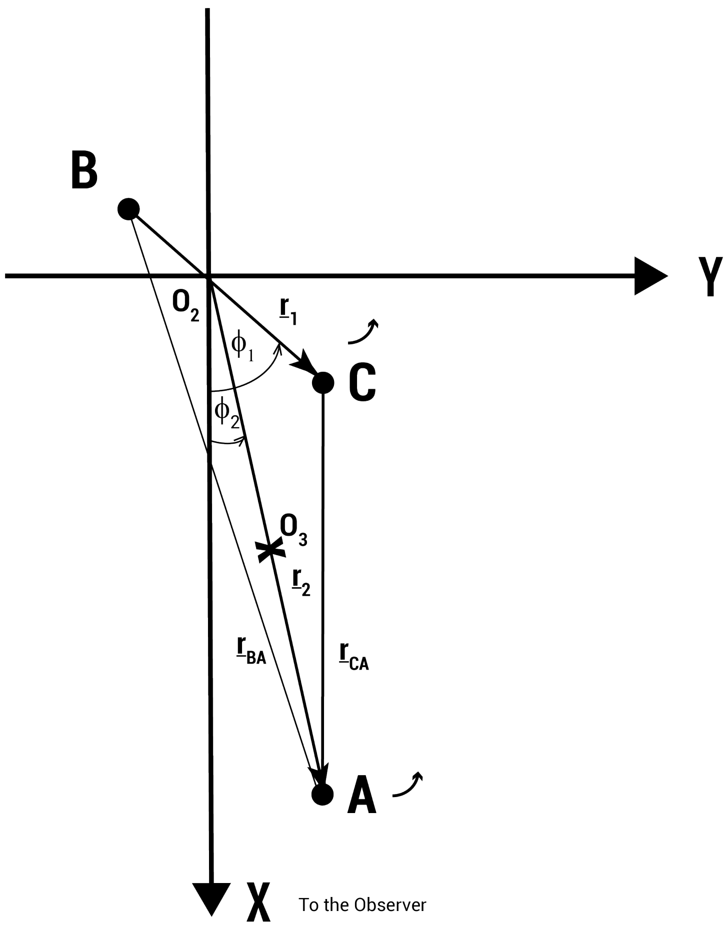
Let $\vec{r}_1$ the radius vector directed from star B toward star C, and $\vec{r}_2$ another radius vector directed from the centre of mass of stars B and C toward star A.

By the use of these two (Jacobian) vectors, the position vectors connecting stars B and C with star A, can be written as:

(2.3) $$\vec{r}_\mathrm{BA} = \vec{r}_\mathrm{A} - \vec{r}_\mathrm{B} = \vec{r}_2 + \frac{m_\mathrm{C}}{m_\mathrm{BC}}\vec{r}_1,$$ (1 p)

(2.4) $$\vec{r}_\mathrm{CA} = \vec{r}_\mathrm{A} - \vec{r}_\mathrm{C} = \vec{r}_2 - \frac{m_\mathrm{B}}{m_\mathrm{BC}}\vec{r}_1.$$ (1 p)

Using the facts that ($i_\mathrm{rel} = 0°$) and, furthermore, $i_1 = i_2 = 90°$, it is worthy to introduce the orbital plane as reference frame. Let the origin of our frame of reference be the centre of mass



of the close binary (denoted as O_2 in figure above). Furthermore, let axis X directed toward the line of sight. Therefore, axis Y is located in the tangential plane of the sky.
In this frame of reference, the components of vectors $\vec{r}_1$ and $\vec{r}_2$ can simply be written as

(2.5) $\vec{r}_1 = a_1 \left[\cos(\phi_1); \sin(\phi_1)\right],$ (1 p)

(2.6) $\vec{r}_2 = a_2 \left[\cos(\phi_2); \sin(\phi_2)\right],$ (1 p)

as ϕ_1 takes the values of $2k\pi$ in the moments, when star C eclipses star B, and similarly, $\phi_2 = 2k\pi$ when star A "eclipses" the centre of mass of the inner binary.
One can also notice that there is a simple relation between these position angles and the photometric phases defined above as

(2.7) $\phi_{1,2} = 2\pi\varphi_{1,2}$ (1 p)

We are interested in the sky-projected distances of the stellar disks, which, in this frame of reference, are equal to the y components of the vector equations (2.3 and 2.4). Accordingly,

(2.8) $d_{\text{A–B}} = \left| a_2 \left[\sin(2\pi\varphi_2) + \frac{m_\text{C}}{m_\text{BC}} \frac{a_1}{a_2} \sin(2\pi\varphi_1) \right] \right|,$ (1 p)

(2.9) $d_{\text{A–C}} = \left| a_2 \left[\sin(2\pi\varphi_2) - \frac{m_\text{B}}{m_\text{BC}} \frac{a_1}{a_2} \sin(2\pi\varphi_1) \right] \right|.$ (1 p)

From this point we can use different combinations of eclipsing events given in the table. Therefore, here we use one possible order only for illustration.

We notice that both outer contacts (i.e. contacts I and IV) of the first event stars A and B are involved. Therefore, we can write the same equation for both contacts as follows:

$$(2.10)\quad \frac{R_\mathrm{A}+R_\mathrm{B}}{a_2} = \left|\sin(2\pi\varphi_2) + \frac{m_\mathrm{C}}{m_\mathrm{BC}}\frac{a_1}{a_2}\sin(2\pi\varphi_1)\right|. \qquad (1\,\mathrm{p})$$

Therefore, we have two independent equations and two unknown variables, namely

$$(2.11)\quad \frac{R_\mathrm{A}+R_\mathrm{B}}{a_2},\ \text{and}\ \frac{m_\mathrm{C}}{m_\mathrm{BC}}\frac{a_1}{a_2} = \frac{a_\mathrm{B}}{a_2}. \qquad (2\,\mathrm{p})$$

The computation is as follows:
For the 1st contact $\sin 2\pi\varphi_2 = -0.158\,296$, $\sin 2\pi\varphi_1 = 0.696\,364$, while for the 4th one $\sin 2\pi\varphi_2 = 0.166\,356$, $\sin 2\pi\varphi_1 = -0.993\,105$. Consequently

$$(2.12)\quad 0.158\,296 - 0.696\,364\,\frac{a_\mathrm{B}}{a_2} = 0.166\,356 - 0.993\,105\,\frac{a_\mathrm{B}}{a_2}, \qquad (1\,\mathrm{p})$$

$$(2.13)\quad \frac{a_\mathrm{B}}{a_2} = 0.027\,161 \qquad (1\,\mathrm{p})$$

and, therefore,

$$(2.14)\quad \frac{R_\mathrm{A}+R_\mathrm{B}}{a_2} = 0.1394. \qquad (1\,\mathrm{p})$$

Now, considering the also available 3rd contact moment of the same event (in which similarly, stars A and B play the roles), one can get

$$(2.15)\quad \frac{R_\mathrm{A}-R_\mathrm{B}}{a_2} = |\sin(2\pi\times 0.0155) + 0.027\,161\times\sin(2\pi\times 0.1743)| \qquad (1\,\mathrm{p})$$

$$(2.16)\quad \frac{R_\mathrm{A}-R_\mathrm{B}}{a_2} = 0.1214 \qquad (1\,\mathrm{p})$$

Combining these results, we obtain that

$$(2.17)\quad \frac{R_\mathrm{A}}{a_2} = \frac{1}{2}\left(\frac{R_\mathrm{A}+R_\mathrm{B}}{a_2} + \frac{R_\mathrm{A}-R_\mathrm{B}}{a_2}\right) = \boxed{0.1304} \qquad (2\,\mathrm{p})$$

$$(2.18)\quad \frac{R_\mathrm{B}}{a_2} = \frac{1}{2}\left(\frac{R_\mathrm{A}+R_\mathrm{B}}{a_2} - \frac{R_\mathrm{A}-R_\mathrm{B}}{a_2}\right) = \boxed{0.0090} \qquad (2\,\mathrm{p})$$

We repeat a very similar calculation for the 3rd contacts of events 3 and 4, in which cases star B is substituted with star C.
For the 3rd contact of the third event $\sin 2\pi\varphi_2 = 0.102\,237$, $\sin 2\pi\varphi_1 = -0.706\,218$, while the 3rd contact of the fourth event $\sin 2\pi\varphi_2 = 0.094\,108$, $\sin 2\pi\varphi_1 = -0.993\,469$.
Therefore,

$$(2.19)\quad 0.102\,237 + 0.706\,218\,\frac{a_\mathrm{C}}{a_2} = 0.094\,108 + 0.993\,469\,\frac{a_\mathrm{C}}{a_2} \qquad (1\,\mathrm{p})$$

$$(2.20)\quad \frac{a_\mathrm{C}}{a_2} = 0.028\,298 \qquad (1\,\mathrm{p})$$

and, accordingly,

$$(2.21)\quad \frac{R_\mathrm{A} - R_\mathrm{C}}{a_2} = 0.1222 \qquad (1\,\mathrm{p})$$

Taking into account that R_A/a_2 is already known from the previous stage, the dimensionless relative radius of star C can be obtained simply as

$$(2.22)\quad \frac{R_\mathrm{C}}{a_2} = \frac{R_\mathrm{A}}{a_2} - \frac{R_\mathrm{A} - R_\mathrm{C}}{a_2} = 0.1304 - 0.1222 = \boxed{0.0082} \qquad (2\,\mathrm{p})$$

The other possibility is, however, that e.g. from the 1st contact of event 4 we calculate the sum $(R_\mathrm{A} + R_\mathrm{C})/a_2$, as

$$(2.23)\quad \frac{R_\mathrm{A} + R_\mathrm{C}}{a_2} = |\sin(2\pi \times 0.9736) - 0.028\,298 \times \sin(2\pi \times 0.6882)|$$

$$(2.24)\quad \frac{R_\mathrm{A} + R_\mathrm{C}}{a_2} = 0.1389,$$

and then we obtain that

$$(2.25)\quad \frac{R_\mathrm{A}}{a_2} = \frac{1}{2}\left(\frac{R_\mathrm{A} + R_\mathrm{C}}{a_2} + \frac{R_\mathrm{A} - R_\mathrm{C}}{a_2}\right) = 0.1306,$$

$$(2.26)\quad \frac{R_\mathrm{C}}{a_2} = \frac{1}{2}\left(\frac{R_\mathrm{A} + R_\mathrm{C}}{a_2} - \frac{R_\mathrm{A} - R_\mathrm{C}}{a_2}\right) = 0.0084.$$

********** Several additional, equivalently acceptable scenarios **********

We can write three equations for $(R_\mathrm{A} + R_\mathrm{B})/a_2$:

$$\frac{R_\mathrm{A} + R_\mathrm{B}}{a_2} = 0.158\,296 - 0.696\,364\,\frac{a_\mathrm{B}}{a_2} \ \text{(1st event Ist contact)}$$

$$\frac{R_\mathrm{A} + R_\mathrm{B}}{a_2} = 0.166\,356 - 0.993\,105\,\frac{a_\mathrm{B}}{a_2} \ \text{(1IV)}$$

$$\frac{R_\mathrm{A} + R_\mathrm{B}}{a_2} = 0.166\,433 - 0.995\,966\,\frac{a_\mathrm{B}}{a_2} \ \text{(2I)}$$

Combining them one can get:

contacts	a_B/a_2	$(R_\mathrm{A} + R_\mathrm{B})/a_2$
1I–1IV	0.027 161	0.1394
1I–2I	0.027 159	0.1394
1IV–2I	0.026 988	0.1396

Similarly, for $(R_\mathrm{A} - R_\mathrm{B})/a_2$:

$$\frac{R_\mathrm{A} - R_\mathrm{B}}{a_2} = 0.097\,235 + 0.889\,001\,\frac{a_\mathrm{B}}{a_2} \ \text{(1III)}$$

$$\frac{R_\mathrm{A} - R_\mathrm{B}}{a_2} = 0.107\,236 + 0.486\,152\,\frac{a_\mathrm{B}}{a_2} \ \text{(5III)}$$

Combining them one can get:

contacts	a_B/a_2	$(R_A - R_B)/a_2$
1III–5III	0.024 826	0.1193

Now for $(R_A + R_C)/a_2$:

$$\frac{R_A + R_C}{a_2} = 0.165\,116 - 0.925\,553\,\frac{a_C}{a_2} \quad (4I)$$

$$\frac{R_A + R_C}{a_2} = 0.150\,847 - 0.414\,376\,\frac{a_C}{a_2} \quad (5IV)$$

The only combination gives:

contacts	a_C/a_2	$(R_A + R_C)/a_2$
4I–5IV	0.027 914	0.1393

Finally, for $(R_A - R_C)/a_2$:

$$\frac{R_A - R_C}{a_2} = 0.115\,353 + 0.185\,529\,\frac{a_C}{a_2} \quad (1II)$$

$$\frac{R_A - R_C}{a_2} = 0.102\,237 + 0.706\,218\,\frac{a_C}{a_2} \quad (3III)$$

$$\frac{R_A - R_C}{a_2} = 0.094\,108 + 0.993\,469\,\frac{a_C}{a_2} \quad (4III)$$

The combinations are:

contacts	a_C/a_2	$(R_A - R_C)/a_2$
1II–3III	0.025 190	0.1200
1II–4III	0.026 295	0.1202
3III–4III	0.028 299	0.1222

********** End additional, equivalently acceptable scenarios **********

Now, we are in the position to calculate the inner mass ratio q_1, as follows:

$$(2.27)\quad q_1 = \frac{a_B/a_2}{a_C/a_2} = \frac{0.027\,161}{0.028\,298} = \boxed{0.9598} \qquad (2\,\mathrm{p})$$

Furthermore, we can also get the ratio of the semi-major axes as

$$(2.28)\quad \frac{a_1}{a_2} = \frac{a_B}{a_2} + \frac{a_C}{a_2} = 0.027\,161 + 0.028\,298 = \boxed{0.055\,459} \qquad (2\,\mathrm{p})$$

The requested results with the range of the full marks (summary):

$$0.1150 \le \frac{R_A}{a_2} \le 0.1450$$

$$0.0075 \le \frac{R_B}{a_2} \le 0.0105$$

$$0.0070 \le \frac{R_C}{a_2} \le 0.0100$$

$$0.045 \le \frac{a_1}{a_2} \le 0.065$$

$$0.80 \le q_1 \le 1.25$$

d) The quickest method to obtain q_2 comes with the double use of Kepler's third law. Writing this law for both the inner and outer orbits, and dividing them one can get, that

$$\left(\frac{a_1}{a_2}\right)^3\left(\frac{P_2}{P_1}\right)^2=\frac{m_{\rm BC}}{m_{\rm ABC}}=\frac{q_2}{1+q_2}. \tag{2.29}$$ (4 p)

Therefore

$$q_2=\frac{\left(\frac{a_1}{a_2}\right)^3\left(\frac{P_2}{P_1}\right)^2}{1-\left(\frac{a_1}{a_2}\right)^3\left(\frac{P_2}{P_1}\right)^2} \tag{2.30}$$ (2 p)

$$q_2\approx\frac{0.055\,459^3\times 50.206\,763^2}{1-0.055\,459^3\times 50.206\,763^2}\approx\boxed{0.7543} \tag{2.31}$$ (2 p)

ii) The circular velocity of star A is

$$v_{\rm A}=\frac{2\pi}{P_2}r_{\rm A}. \tag{2.32}$$ (1 p)

The amplitude of the RV curve is equal to maximum value of the line-of-sight component of that velocity, i.e.

$$K_{\rm A}=v_{\rm A}\sin i_2. \tag{2.33}$$ (1 p)

The sinusoidal component in the occurrence of the eclipsing minima times comes from the light-travel time effect caused by the revolution of stars B and C around the centre of mass of the whole triple system. In this regard the motion of components B and C can simply be substituted by the movement of their centre of mass along the outer orbit. Therefore, the total variation of the inner binary's distance to the Earth is the line-of-sight component of the diameter of the orbit of the centre of mass of the inner binary around the centre of mass of the complete triple system, i.e.

$$\Delta z_{\rm BC}=2r_{\rm BC}\sin i_2. \tag{2.34}$$ (2 p)

Therefore, the amplitude of the light-travel time sine wave is

$$A_{\rm ETV}=\frac{\Delta z}{2c}=\frac{r_{\rm BC}\sin i_2}{c}\rightarrow r_{\rm BC}=\frac{cA_{\rm ETV}}{\sin i_2}. \tag{2.35}$$ (2 p)

Such a way, using that

$$q_2=\frac{m_{\rm BC}}{m_{\rm A}}=\frac{r_{\rm A}}{r_{\rm BC}}, \tag{2.36}$$ (1 p)

one can obtain, that

$$q_2=\frac{P_2K_{\rm A}}{2\pi cA_{\rm ETV}}. \tag{2.37}$$ (1 p)

which gives a second, independent determination of the outer mass ratio q_2.

The uncertainty can be estimated either as

$$\Delta q_2=q_2\sqrt{\left(\frac{\Delta P_2}{P_2}\right)^2+\left(\frac{\Delta K_{\rm A}}{K_{\rm A}}\right)^2+\left(\frac{\Delta A_{\rm ETV}}{A_{\rm ETV}}\right)^2} \tag{2.38}$$

or

(2.39) $$\Delta q_2 = q_2\left(\left|\frac{\Delta P_2}{P_2}\right| + \left|\frac{\Delta K_A}{K_A}\right| + \left|\frac{\Delta A_{ETV}}{A_{ETV}}\right|\right)$$ (2 p)

With numerical values:

$$q_2 = \boxed{0.621 \pm 0.047} \text{ or } q_2 = \boxed{0.621 \pm 0.048}$$ (1 p)

Then, the semi-major axis of the outer orbit can be calculated in the following alternative ways:

(2.40) $$a_2 = r_A \frac{1+q_2}{q_2} = \frac{P_2}{2\pi}\frac{K_A}{\sin i_2}\frac{1+q_2}{q_2}$$

(2.41) $$a_2 = r_{BC}(1+q_2) = c\frac{A_{ETV}}{\sin i_2}(1+q_2)$$

(2.42) $$a_2 = r_A + r_{BC} = \frac{P_2}{2\pi}\frac{K_A}{\sin i_2} + c\frac{A_{ETV}}{\sin i_2},$$

One of the three equations above: (2 p)

We assumed that $i_2 = 90°$, therefore $\sin i_2 = 1$. Then

$$a_2 = \boxed{(60.712 \pm 2.851) \times 10^6\,\text{km} = (87.293 \pm 4.099)\,\text{R}_\odot = (0.406 \pm 0.019)\,\text{AU}}$$ (2 p)

The uncertainties from the three different formulae above:

(2.43) $$\Delta a_2 = a_2\sqrt{\left(\frac{\Delta P_2}{P_2}\right)^2 + \left(\frac{\Delta K_A}{K_A}\right)^2 + \left(\frac{\Delta q_2}{q_2(1+q_2)}\right)^2}$$

$$\Delta a_2 \approx \boxed{2.851 \times 10^6\,\text{km} \approx 4.10\,\text{R}_\odot \approx 0.019\,\text{AU}}$$

(2.44) $$\Delta a_2 = a_2\sqrt{\left(\frac{\Delta A_{ETV}}{A_{ETV}}\right)^2 + \left(\frac{\Delta q_2}{1+q_2}\right)^2}$$

$$\Delta a_2 \approx \boxed{4.946 \times 10^6\,\text{km} \approx 7.11\,\text{R}_\odot \approx 0.033\,\text{AU}}$$

(2.45) $$\Delta a_2 = \sqrt{\left(\frac{\Delta P_2}{2\pi}K_A\right)^2 + \left(\frac{P_2}{2\pi}\Delta K_A\right)^2 + (c\Delta A_{ETV})^2}$$

$$\Delta a_2 \approx \boxed{2.849 \times 10^6\,\text{km} \approx 4.10\,\text{R}_\odot \approx 0.019\,\text{AU}}$$

One of the three formulae above with its uncertainty: (3 p)

In what follows, we use the smallest uncertainties (first row above) but the others, and also the nonquadratic ones are acceptable, too.

The total mass of the triple now can be calculated directly from Kepler's third law or, as an alternative way, one can obtain it from the equations on the centripetal accelerations.

(2.46) $$r_A\frac{4\pi^2}{P_2^2} = \frac{Gm_{BC}}{a_2^2}$$

(2.47) $r_{\rm BC}\,\dfrac{4\pi^2}{P_2^2} = \dfrac{Gm_{\rm A}}{a_2^2},$

where the only unknown quantities are the masses, so they can be calculated separately from the two equations or, summing the masses we can get back (formally) Kepler's third law:

(2.48) $m_{\rm ABC} = \dfrac{4\pi^2}{G}\,\dfrac{a_2^3}{P_2^2} = (4.312 \pm 0.607)\,{\rm M}_\odot,$ (2 p)

from which

(2.49) $m_{\rm A} = \dfrac{m_{\rm ABC}}{1+q_2} = \boxed{(2.660 \pm 0.383)\,{\rm M}_\odot}$ (1 p)

(2.50) $m_{\rm BC} = m_{\rm ABC}\,\dfrac{q_2}{1+q_2} = \boxed{(1.652 \pm 0.245)\,{\rm M}_\odot}$ (1 p)

iii) We obtained the total mass ($m_{\rm BC}$) of the inner binary in the second part of the problem. Combining this with the inner mass ratio (q_1) obtained in the first part, one can get that

(2.51) $m_{\rm B} = \dfrac{m_{\rm BC}}{1+q_1} = \dfrac{1.652\,{\rm M}_\odot}{1+0.960} = \boxed{0.843\,{\rm M}_\odot}$ (3 p)

and

(2.52) $m_{\rm C} = q_1 \times m_{\rm B} = 0.960 \times 0.843\,{\rm M}_\odot = \boxed{0.809\,{\rm M}_\odot}$ (3 p)

The dimensionless radius of each star relative to the semi-major axis was calculated in the first part, while the semi-major axis of the outer orbit (a_2) was obtained in the second part. Their multiplication gives the physical radii of the stars:

(2.53) $R_{\rm A} = 0.1304 \times 87.29\,{\rm R}_\odot = \boxed{11.381\,{\rm R}_\odot}$ (3 p)

(2.54) $R_{\rm B} = 0.0090 \times 87.29\,{\rm R}_\odot = \boxed{0.786\,{\rm R}_\odot}$ (3 p)

(2.55) $R_{\rm C} = 0.0082 \times 87.29\,{\rm R}_\odot = \boxed{0.712\,{\rm R}_\odot}$ (3 p)