SOLUTIONS

Part 1.

1.1) From the plot on the right we can get the equation of the straight line as

$$y = -0.76 \cdot x + 7.32$$ [3 points]

which taking to the power of 10 is equivalent to: $f_g = 10^{7.32} M_*^{-0.76}$, being f_g the gas fraction. Multiplying by the stellar mass we finally get:

$$M_{gas} = 10^{7.32} \times M_*^{0.24}$$

a = 10^7.32 = 2.09 E7 [1 point] ; b = 0.24 [1 point]

No tolerance allowed for these values

1.2) [15 points]

V	LogV	K	Zx	Zy	ZxZy
79.4	1.8998	-16.8	-1.1778	1.3873	-1.6339
100.1	2.0004	-19.2	-0.7831	0.4970	-0.3892
158.5	2.2000	-21.3	-0.0001	-0.2819	0.0000
251.2	2.4000	-21.4	0.7845	-0.3190	-0.2502
316.2	2.5000	-24	1.1765	-1.2834	-1.5100
Ave	2.2001	-20.54		sum ZxZy	-3.7833
St. Dev	0.2549	2.696		r	-0.9458

The best-fit straight line is $K = -10 \cdot log_{10}(V_{max}) + 1.47$.

A correct answer does not need to show step-by-step calculations.
Tolerance: +/- 0.01.
Partial score if the full answer is not correct: Slope: [3 points] ; Intercept: [2 points] ; calculations: if every single number is right except the final answer [10 points] ; if at least half the numbers are correct [5 points]. Tolerance in each number: +/- 0.01

Part 2.

2.1) [10 points]

Galaxies are at the same distance, therefore the difference in apparent magnitudes is the same as in absolute magnitudes, and this relates to a luminosity ratio:

$$(k_1 - k_2) = (K_1 - K_2) = -2.5\times log_{10}(\frac{L_1}{L_2})$$ [2 points]

For having the same stellar populations, and ignoring interstellar extinction, the luminosity of each galaxy is proportional to its stellar mass with the same mass-to-light ratio for both galaxies, such that:

$$(k_1 - k_2) = -2.5\times log_{10}(\frac{M_{*1}}{M_{*2}})$$ [2 points]

The difference in magnitudes being 6, we obtain:

$$(\frac{M_{*1}}{M_{*2}}) = 10^{2.4}$$ [2 points]

No tolerance allowed in the exponent value.

2.2) [4 points]

Using the last result in combination to that of 1A:

$$\frac{M_{gas1}}{M_{gas2}} = \left(\frac{M_{*1}}{M_{*2}}\right)^{0.24}$$ [2 points]

$$\frac{M_{gas1}}{M_{gas2}} = 10^{0.576}$$ [2 points]

Exponent approximated to 2 digits is OK.

2.3) [6 points]

We found the slope of the TF relation in 1B: $\frac{\Delta K}{\Delta \log(V_{max})} = 10$, with $\Delta K = 6$ for our galaxies: $\frac{V_{max1}}{V_{max2}} = 10^{0.6}$ [2 points]. Now consider the mass distribution as approximately spherically-symmetric, giving:

$$V_{max} = \sqrt{\frac{G\cdot M_{tot}(<R_{max})}{R_{max}}},$$ [2 points]

which combined with the ratio just found two lines above leads to:

$$\frac{M_{tot1}}{M_{tot2}} = 10^{1.2}$$ [2 points]

No tolerance allowed in the exponent value.

Part 3. [15 points]

Baryonic mass is $M_{Baryonic} = \frac{4.39\times10^{11}}{7.82}\ [M_\odot]$ and Dark matter mass is given by $M_{dm} = 6.82 \times \frac{4.39\times10^{11}}{7.82}\ [M_\odot]$. The baryonic contribution needs to be decomposed into stellar and gaseous masses, which is not doable analytically but must be faced numerically, by tuning the numbers, knowing their sum and knowing the relation between them from 1.1. The final table looks like this:

Galaxy	Apparent magnitude k	$M_{gas}\ [M_\odot]$	$M_*\ [M_\odot]$	$M_{dm}\ [M_\odot]$	$M_{tot}\ [M_\odot]$
G_1	19.2	7.67×10^{9}	4.85×10^{10}	3.83×10^{11}	4.39×10^{11}

Equation to be solved: $1 - k = 0.1415\, k^{0.24}$ This gives k = 0.8634

5 points for each value, with a tolerance of 5%

If does not score for Mg, M*, but has a correct equation , 5 points

Part 4

4.1) [4 points]

We basically found in 2.1 that $(\frac{M_{*1}}{M_{*2}}) = 10^{\frac{\Delta K}{-2.5}}$, being $\Delta K = -6$. In this new case, we have the extreme values of ΔK to be -6.4 and -5.6, therefore the extreme values for the exponent are these values divided by -2.5, then $e \in [2.24, 2.56]$

No tolerance allowed in limits of the interval.

4.2) [10 points]

We have to invert the equation from 1.2 to get: $log_{10}(V_{max}) = -\frac{K}{10} + 0.1467$. Now we can get the difference between

the measured values and linear-fit predictions, approximating numbers to the second digit:

$K[mag]$	$V_{max}\ [km/s]$	$log_{10}(V_{max})$ measured	$log_{10}(V_{max})$ from linear fit	$\lvert\Delta\ log_{10}(V_{max})\rvert$
-16.8	79.4	1.9	1.83	0.07
-19.2	100.1	2.0	2.07	0.07
-21.3	158.5	2.2	2.28	0.08
-21.4	251.2	2.4	2.29	0.11
-24.0	316.2	2.5	2.55	0.05

Taking two times the RMS of the residuals: $\sigma_{stat} = 0.157$

A correct answer does not need to show step-by-step calculations.
If the final value is not correct, up to [5 points] for partial calculations: [1 point] for each value of $log_{10}(V_{max})$ correctly calculated.

Tolerance in each number: +/- 0.01

4.3) With the uncertainties, the expression $log_{10}(V_{max}) = -\frac{K}{10} + 0.1467$ becomes:

$log_{10}(V_{max}) = -\frac{K}{10} + 0.1467 \pm \frac{0.2}{10} \pm 0.157$ [2 points]

Values approximated to the second digit are accepted.

Writing the same expression down for galaxy 1 and 2, subtracting, and using $\Delta K = -6$ we get:

$log_{10}(\frac{V_{max1}}{V_{max2}}) = 0.6 \pm 0.02 \pm 0.157 \pm 0.02 \pm 0.157$ [4 points]

Which leads to the extreme values: 0.954 and 0.246. [2 points]

Finally remembering that circular velocities scale as the square root of the total mass, we have to multiply this exponent by 2, getting the final answer:

$$g \in [\,0.49,\ 1.91]$$ [2 points]

A tolerance of +/- 0.02 is acceptable.